An inner scale for dissipation of helicity in turbulence

P. D. Ditlevsen[1] and P. Giuliani[2]

[1]Niels Bohr Institute, Geophysical Department
Juliane Maries Vej 30, DK-2100 Copenhagen O, Denmark
[2]Dipartimento di Fisica and Istituto Nazionale di Fisica della Materia
Università della Calabria, 87036 Rende (CS), Italy

Contact e-mail: pditlev@gfy.ku.dk

1 Helical turbulence

The existence of a second quadratic inviscid invariant, the helicity, besides the energy, in a helical turbulent flow leads to coexisting cascades of energy and helicity [1]. An equivalent of the four-fifth law for the longitudinal third order structure function, which is derived from energy conservation, is easily derived from helicity conservation [2, 3]. This is a scaling relation for the third order correlator associated with the spectral flux of helicity, $\langle \delta \mathbf{v}_{\parallel}(l) \cdot [\mathbf{v}_{\perp}(r) \times \mathbf{v}_{\perp}(r+l)] \rangle = (2/15)\overline{\delta} l^2$, where $\overline{\delta}$ is the mean dissipation of helicity. This relation is called the 'two-fifteenth law' due to the numerical prefactor. The two-fifteenth law establishes another non-trivial scaling relation for velocity differences in a turbulent helical flow.

The ratio of dissipation of helicity to dissipation of energy in spectral space is proportional to the wave-number k. This is leading to a different inner – or Kolmogorov scale for helicity than for energy [5]. The Kolmogorov scale η for energy dissipation is obtained from $\overline{\varepsilon} \sim \delta u_\eta^3/\eta \sim \nu \delta u_\eta^2/\eta^2 \Rightarrow \eta \sim (\nu^3/\overline{\varepsilon})^{1/4}$, where δu_l is a typical variation of the velocity over a scale l and $\overline{\varepsilon}$ is the mean energy dissipation. The Kolmogorov scale ξ for dissipation of helicity is obtained by balancing the helicity dissipation and the spectral helicity flux. With dimensional counting we have $\overline{\delta} \sim \nu \delta u_\xi^3/\xi^2$ and using $\delta u_l \sim (l\overline{\varepsilon})^{1/3}$ we obtain the inner scale ξ for helicity dissipation,

$$\xi \sim (\nu^3 \overline{\varepsilon}^2/\overline{\delta}^3)^{1/7}, \tag{1}$$

where ν is the kinematic viscosity, $\overline{\varepsilon}$ the mean energy dissipation and $\overline{\delta}$ the mean helicity dissipation. The inner scale for helicity is always larger than the Kolmogorov scale for energy so in the high Reynolds number limit the flow will always be helicity free in the small scales, much in the same way as the flow will be isotropic and homogeneous in the small scales. However, as is the case for the enstrophy, we must have a blow up of helicity for high Reynolds number flow in order to permit the energy to cascade to the dissipation scale since the spectral helicity density dimensionally dominates with a factor k over the spectral energy density. Helicity is, in contrast to enstrophy, an inviscid invariant in 3D turbulence so the only way there can be a blow up of helicity is if there is a detailed balance between positive and negative helicity production in the range $K_H < k < K_E$. So the situation in 3D turbulence with cascades of energy and helicity is very different from the situation in 2D turbulence since helicity is not a positive quantity. In 2D turbulence, where enstrophy is a positive inviscid invariant, the cascade of enstrophy prohibits a forward cascade of energy.

2 Model simulation

The idea is illustrated in a shell model of turbulence. Shell models are toy-models of turbulence which by construction have second order inviscid invariants similar to energy and helicity in 3D turbulence. The advantage of shell models is that they can be investigated numerically for very high Reynolds numbers, in contrast to the Navier-Stokes equation. Shell models lack any spatial structures so we stress that only certain aspects of the turbulent cascades have meaningful analogies in the shell models. This should especially be kept in mind when studying helicity which is intimately linked to spatial structures, and the dissipation of helicity to reconnection of vortex tubes [4]. So the following only concerns the spectral aspects of the helicity and energy cascades. The most well studied shell model, the GOY model [6, 7], is defined from the governing equation,

$$\dot{u_n} = ik_n(u_{n+2}u_{n+1} - \frac{\epsilon}{\lambda}u_{n+1}u_{n-1} + \frac{\epsilon-1}{\lambda^2}u_{n-1}u_{n-2})^* \\ -\nu k_n^2 u_n + f_n \tag{2}$$

with $n = 1, ..., N$ where the u_n's are the complex shell velocities. The wave-numbers are defined as $k_n = \lambda^n$, where λ is the shell spacing. The second and third terms are dissipation and forcing. The model has two inviscid invariants, energy, $E = \sum_n E_n = \sum_n |u_n|^2$, and 'helicity', $H = \sum_n H_n = \sum_n (\epsilon-1)^{-n}|u_n|^2$. The model has two free parameters, λ and ϵ. The 'helicity' only has the correct dimension of helicity if $|\epsilon - 1|^{-n} = k_n \Rightarrow 1/(1-\epsilon) = \lambda$. In this work we use the standard parameters $(\epsilon, \lambda) = (1/2, 2)$ for the GOY model.

Figure 1 shows two shell model simulations, one with a helicity free forcing and one with coexisting cascades of energy and helicity. The spectral fluxes of energy and helicity are plotted against wave-number. The helicity flux (diamonds) in the case of a helical forcing shows a crossover between a regime with a constant cascade of helicity and a regime dominated by balanced dissipation of positive and negative helicity (for even – and odd numbered shells). The scaling in this regime is governed by the dissipation of helicity $D_n \sim \nu k_n^3 |u_n|^2 \sim \nu k_n^{7/3}$.

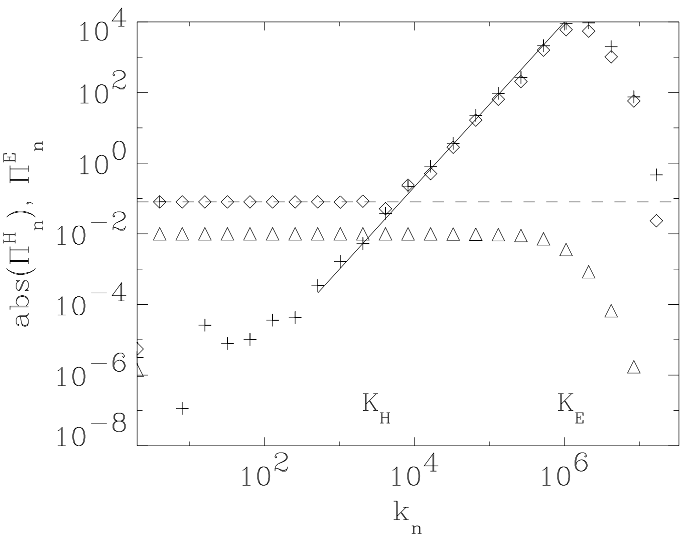

Figure 1: The absolute values of the helicity flux $|\langle \Pi_n^H \rangle|$ (diamonds) show a crossover from the inertial range for helicity to the range where the helicity is dissipated. The line has a slope of 7/3 indicating the helicity dissipation. The dashed lines indicate the helicity input $\overline{\delta}$. The crosses is the helicity flux in the case $\overline{\delta} = 0$ where there is no inertial range and K_H coincides with the integral scale. The triangles are the energy flux $\langle \Pi_n^E \rangle$.

The crossover defines the inner scale $\xi = K_H^{-1}$ for helicity dissipation. In the first regime both the four-fifth – and the two-fifteenth' law applies, in the second regime only the four-fifth law applies. The position of the inner scale K_H depends on the input of helicity $\overline{\delta} = 0$. Figure 2 shows a set of simulations performed with different helicity inputs. When scaling the wave number with K_H and the helicity flux with $\overline{\delta} = 0$ a clear data collapse between the different simulations is seen, confirming the scaling (1).

3 Conclusions

The role of helicity in 3D turbulence is different from the role of enstrophy in 2D turbulence. In 3D helical turbulence the helicity is dissipated within the inertial range of energy cascade. Thus there exist two inertial ranges in helical turbulence, a range smaller than K_H with coexisting cascades of energy and helicity where both the four-fifth - and the two-fifteenth law applies, and a range between K_H and K_E where the flow is non-helical and only the four-fifth law

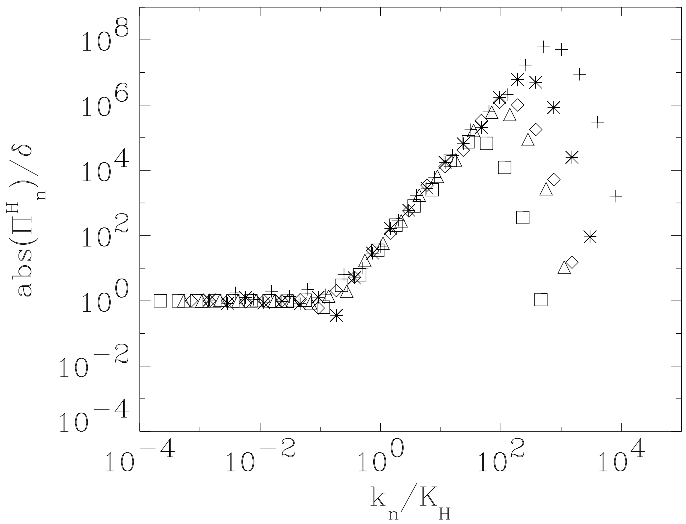

Figure 2: Five simulations with constant viscosity $\nu = 10^{-9}$, constant energy input $\overline{\varepsilon} = 0.01$ and varying helicity input $\overline{\delta} = (0.0001, 0.001, 0.005, 0.01, 0.08)$ are shown. The absolute values of the helicity flux $|\langle \Pi_n^H \rangle|$ divided by $\overline{\delta}$ is plotted against the wave number divided by $K_H = (\nu^3 \overline{\varepsilon}^2 / \overline{\delta}^3)^{-1/7}$, which is obtained from (1) neglecting $O(1)$ constants.

applies. In this range there is a detailed balance between positive and negative helicity associated with the structures where the energy is dissipated.

References

[1] M. Lesieur, 'Turbulence in Fluids', Third edition, Kluwer Academic Publishers, 1997.

[2] V. S. L'vov et al. chao-dyn/9705016 (unpublished).

[3] O. Chkhetiani, JETP Lett., 63, 808, 1996.

[4] E. Levich, L. Shtilman and A.V. Tur, Physica A, 176, 241, 1991.

[5] P. D. Ditlevsen and P. Giuliani, chao-dyn/9910013.

[6] E. B. Gledzer, Sov. Phys. Dokl, 18, 216, 1973.

[7] L. Kadanoff et al., Phys. Fluids, 7, 617, 1995.