Structural and stochastic transitions in the climate

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Summary. The cause for and possible prediction of rapid climate changes is poorly understood. The most pronounced changes observed, beside the glacial terminations, are the Dansgaard-Oeschger (DO) events. Present day general circulation climate models simulating glacial conditions are not capable of reproducing these rapid shifts. It is thus not known if they are due to bifurcations in the structural stability of the climate or if they are induced by stochastic fluctuations. From analysis of a high resolution ice core record the bifurcation scenario can be excluded. This strongly suggests that they are noise induced and thus have very limited predictability.

The DO climate events in the last glacial period, shown in figure 1, are observed in a variety of paleoclimatic records with an almost global extend. They are abrupt jumps between a distinct cold climatic state (the stadial) and a warm state (the interstadial). By time scale separation the climate dynamics can be split into a slow component and a fast component. The dynamics is then described as an effective non-linear stochastic proces in which the fast chaotic variations are treated as a stochastic noise forcing the slow climate dynamics[1] This description is, even in the linear approximation, very successful in explaining the red noise spectra observed in climate records[2]The observation that the extremely complex dynamics of the climate system seems to exhibit bifurcations, which are usually associated with low order non-linear dynamical systems, is remarkable.

It is thus a fundamental question if the rapid climate changes are due to bifurcations, where the system becomes structurally unstable as a function of some control parameter, or if the jumps between two quasi-stable states are purely stochastic and noise induced. At present we do not have final theories for which scenario describes the observed rapid climate changes. On the contrary, climate models in favor of both scenarios or even induced - or self-sustained oscillations have been proposed. The goal here is then to extract enough information from the observations to discriminate between the different possible scenarios. The observations to be analyzed are the new high temporal resolution NGRIP ice-core.

The two generic characteristics of the approach to a bifurcation point in a noisy system are increased variance of the observed signal, following from the fluctuation-dissipation theorem and the corresponding increased autocorrelation related to critical slow down. In order to identify a bifurcation from observations the two signals; increased variance and increased autocorrelation, should be detectable. The dynamics of the effective variable x are governed by the Langevin

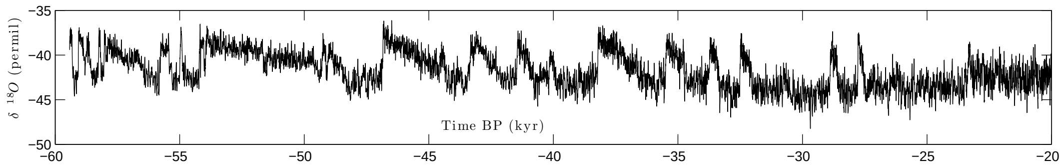

Figure 1: The NGRIP isotope record is a climatic temperature proxy showing rapid climate shift between two states in the glacial climate[3].

equation, $\dot{x} = -\partial_x U_\mu(x) + \sigma\eta$, where $U_\mu(x)$ is a double-well potential, η is a white noise and σ is the intensity of the noise. For small noise intensity we may expand around the relevant minimum, x_0, (taken to be 0 for convenience): $U_\mu(x) = U_\mu(0) + \alpha_\mu x^2/2$. Then equation (1) becomes the linear Ornstein-Uhlenbeck process: $\dot{x} = -\alpha_\mu x + \sigma\eta$. The fluctuation-dissipation theorem then gives $\langle x^2 \rangle = \sigma^2/(2\alpha_\mu)$. If a bifurcation point is approached typically a local maximum and a local minimum in the potential $U_\mu(x)$ merge, such that the local minimum disappears and the system jumps into another stable state (local minimum of $U_\mu(x)$). In this process we have $\alpha_\mu \to 0$ for $\mu \to \mu_0$. Thus the variance grows as the bifurcation point is approached. Likewise the autocorrelation, given as $c(t) = \exp(-\alpha_\mu |t|)$, will have an increasing correlation time $T = 1/\alpha_\mu$ as the bifurcation point is approached. This is the phenomenon of critical slow down. The ratio $\langle x^2 \rangle / T = \sigma^2/2$ is a constant, thus if an increased autocorrelation is seen in a data series without increase in the variance, this cannot be seen as a sign of a forthcoming bifurcation.

In order to investigate the significance in detection of a bifurcation or tipping point from a data series, two simulations of the Langevin equation (1) with a double well potential $U_\mu(x) = x^4/4 - x^2/2 - \mu x$ are performed. In the first the control parameter $\mu(t)$ is changing linearly with time, such that the bifurcation point $\mu_0(= -2\sqrt{3}/9)$ is reached at time $t = 900$ time units; $\mu(t) = \mu_0 * t/900$. A realization is shown in figure 2(a) with $\sigma = 0.1$. Note that the system jumps at some time prior to the bifurcation, since the potential barrier becomes small in comparison to the intensity of the noise. In the other case (figure 2(b)) the parameter $\mu = 0$ is kept constant. This simulation is run for a long time, with $\sigma = 0.25$, until a purely noise induced jump from the one steady state to the other occurs. The time is then reset to zero 900 time units prior to the jump. In the first scenario variance and autocorrelation time increase prior to the jump, while in the second scenario this is not the case. The red curves show the steady states as a function of time. We want to be able to distinguish between these two scenarios prior to the jump[4]. Especially, in the first case we want to be able to distinguish between a true

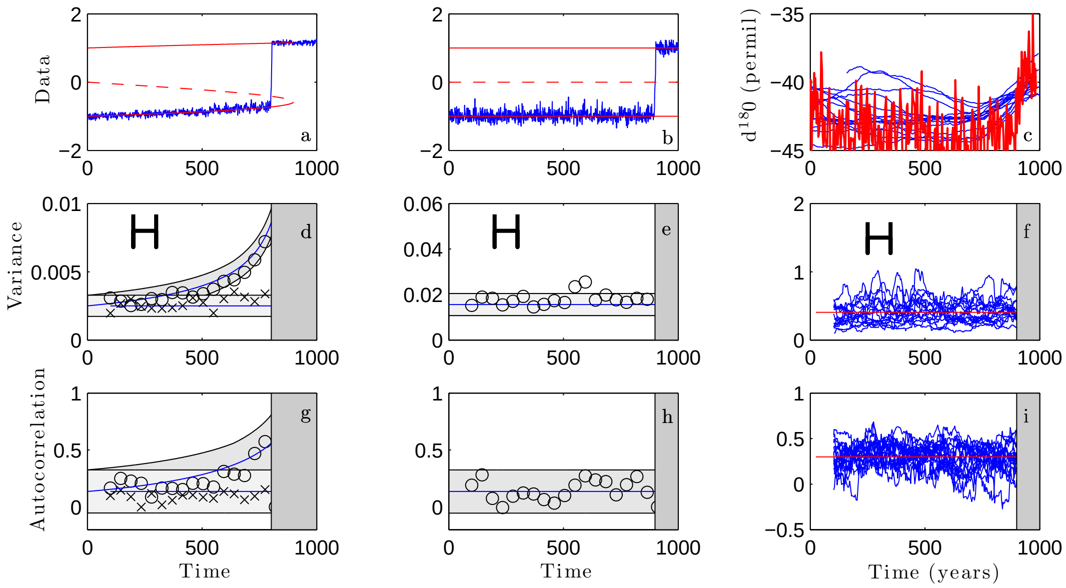

Figure 2: The two first top panels shows realizations of stochastic motion in a double-well potential. In the first case (a) a bifurcation point is approached in time, where the residing stable state disappears and the system jumps into the other remaining stable state. The jump happens some short time before the true bifurcation, where the potential barrier is small in comparison to the noise intensity. In (b) the potential is static, there is no bifurcation and the jump from the lower - to the upper stable state is purely noise driven. The red curves shows the steady states as functions of time. Top panel (c)shows 25 DO events, including the last termination, aligned such that the transitions all occur at the same time. The blue curves are 100 years smoothed records, the red curve is the approximately 5 year resolution of the (randomly chosen) DO4. Middle and bottom panels show the variances and autocorrelations calculated in running windows indicated by the black bars. The observations (right) are consistent with the purely noise induced transitions (middle). See text for explanations.

warning and a false alarm due to a coincidental fluctuation in variance $\langle x^2 \rangle$ and autocorrelation $\langle x(t)x(t+1)\rangle/\langle x^2\rangle$. This is done in the first column for the variance (figure 2:d) and autocorrelation (figure 2:g) calculated within a running window of 100 time units, as indicated by the black bar. The scenario is compared to the steady state scenario, corresponding to $\mu = 0$ and $\sigma = 0.1$, where no jumps occur (crosses).

The analytic values for the two scenarios are plotted as blue curves (a constant in the no-jump scenario). The gray bands are the two-sigma (2Σ) levels for the calculated variance and autocorrelation within the given window size. This uncertainty (denoted Σ, not to be confused with σ) is given as $\Sigma = \Sigma_0/\sqrt{n}$, where n is the number of independent measurements in the window and Σ_0 is a constant determined by simulation. Obviously, in the second scenario with $\sigma = 0.25$ in the right column, there will be no early warning. In the first column we see that the detection of increased variance is more significant than the detection of increased autocorrelation. The window size chosen is a trade-off between a short window with too large two-sigma bands and a too long window for which the bias from the signal being non-stationary within the window becomes too large.

Figure 2:c shows 17 DO events after 60 kyr B2k, dated by annual layer counting aligned such that the transitions all begin at $t = 900$ years. The blue curves are 100 years smoothed records. The red curve is the approximately 1 year resolution of a randomly chosen event (DO4). Figure 2:f is the running variance calculated from each of the high-resolution transitions. The length of the window is indicated by the black bar, the red line is the mean. Figure 2:i is the corresponding autocorrelation. Both are calculated in exactly the same way as in the model data.

None of the transitions show any (significant) sign of increased variance and autocorrelation prior to the jumps. This strongly suggest that the jumps are not caused by the approach to a bifurcation point. The finding suggests that internal noise (short time scale fluctuations) is the driver for these climate jumps, this implies that they will not be predictable until they actually are about to happen. This is consistent with the finding that the observed waiting time distribution between consecutive events is well fitted by an exponential, corresponding to a memory-less Poisson process[5].

References

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