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Tipping points: Early warning and wishful thinking

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[1] The causes for and possible predictions of rapid climate changes are poorly understood. The most pronounced changes observed, beside the glacial terminations, are the Dansgaard-Oeschger events. Present day general circulation climate models simulating glacial conditions are not capable of reproducing these rapid shifts. It is thus not known if they are due to bifurcations in the structural stability of the climate or if they are induced by stochastic fluctuations. By analyzing a high resolution ice core record we exclude the bifurcation scenario, which strongly suggests that they are noise induced and thus have very limited predictability.
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[2] Climate changes and especially the risk of rapid and irreversible changes are of great socioeconomic concern [*Intergovernmental Panel on Climate Change*, 2007; *Lenton et al.*, 2008]. Abrupt transitions from one statistically steady state to another occur in many complex dynamical systems [*Scheffer et al.*, 2009]. Common among these is that crossing a critical threshold can lead to a structural change of the system. This scenario is mathematically described as a bifurcation [*Arnold*, 1994], which gives the hope that the generic dynamical behavior at bifurcation- or tipping points may be observed even with only imperfect knowledge of the dynamics of the system. It would be especially useful if early warning signals prior to a climate transition could be identified, and the transition perhaps even prevented.

[3] The two generic characteristics of the approach to a bifurcation point are increased variance of the observed signal, following from the fluctuation-dissipation theorem [*Kubo*, 1966] and the corresponding increased autocorrelation, related to critical slow down. These two signals are connected, and the detection of only one and not the other, cannot be taken as a sign of an approaching tipping point. This is contrary to what was recently claimed [*Dakos et al.*, 2008; *Scheffer et al.*, 2009].

[4] In order to analyze the dynamical behavior prior to a climate transition, we need some sort of definition of what should be considered a transition. In terms of possible bifurcations, transitions between distinct (quasi-)stationary states should be observed. In the case that we are only guided by observations, without theoretical considerations predicting multiple climate states, we will be cautious in identifying rapid changes as transitions. We suggest as a pragmatic criterion that repeated transitions between the two states should be observed. We can not, however, exclude the possibility that a single irreversible transition happens, in a way that the system never returns to the original quasi-stable state. This scenario has been proposed for the Archean rapid increase of atmospheric oxygen (GOE) [*Goldblatt et al.*, 2006]. Other more localized observed abrupt climate changes, such as desertification of North African as analyzed by *Dakos et al.* [2008] and other phenomena with a ramp-like temporal evolution, are probably also single event transition from one stable state to another. The most clearly observed climatic transitions fulfilling our criterion, beside the glacial interglacial transitions themselves, are the Dansgaard-Oeschger events [*Dansgaard et al.*, 1993] in the last glacial period. These are observed in a variety of paleoclimatic records with an almost global extend [*EPICA Community Members*, 2004; *Shackleton et al.*, 2000; *Wang et al.*, 2001], and they reproduce jumps between a distinct cold state (the stadial) and a warm state (the interstadial). The jumps observed in a noisy high resolution record can be objectively detected and dated by a first up-crossing procedure [*Ditlevsen et al.*, 2005]. Other phenomena like the little ice age, the 1930s dustbowl in the US or the 2003 European heat wave (in descending order of duration) are most likely persistent anomalies not associated with jumps between climatic states.

[5] We shall in the following investigate the detectability of the two signals; increased variance and increased autocorrelation. The assessment of the statistical significance for detection is obtained by Monte-Carlo simulations. This is then applied to the new high temporal resolution measurements of the Dansgaard-Oeschger events recorded in the NGRIP ice-core [*North GRIP Members*, 2004]. The conclusion drawn is that these most probably are not generated by bifurcations: They are noise induced transitions without early warning signals. Thus it is necessary to understand the full non-linear structure of the climate system, including assessing the influence by an external perturbation (such as increased greenhouse gas concentrations) on the short time scale fluctuations (noise), which might push the system into a different (quasi-)stationary state.

[6] The simplest bifurcation structure of a dynamical system (Figure 1, left), shows the steady state values of a state variable x characterizing the system: The state variable x could be (a) the meridional overturning circulation (MOC) in the Atlantic ocean [*Broecker*, 1997], (b) the ice volume in the glacial ice sheet [*Calov and Ganopolski*, 2005; *Ditlevsen*, 2009] or (c) the global mean surface temperature [*Budyko*, 1969]. The steady states of the state variable are plotted as a function of a control parameter μ which determine the dynamics of the system. In the three cases the control parameter could be: (a) the freshwater added to the Atlantic ocean at 50–60 N latitude, (b) the summer solstice insolation at 65 N, (c) the atmospheric CO_2 concentration. By changing the control parameter the system goes through a bifurcation (at $\mu = \mu_0$) (Figures 1, top right and 1, bottom

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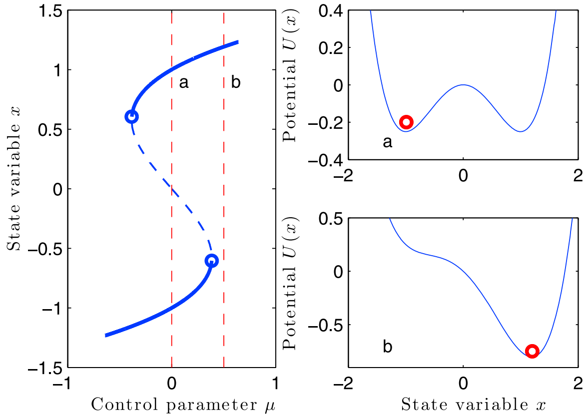

Figure 1. (left) A typical bifurcation diagram with two bifurcation points, marked with blue circles. The two fat branches of the curve are the stable states as a function of a control parameter. The dashed branch in between is the separating unstable fixed point. The vertical red lines indicate where the potentials plotted on the right are taken. (Imagine these as plots in and out of the paper along the red lines a and b). The red dots indicate the state of the system (top right) before and (bottom right) after the jump by a bifurcation.

right). In a system governed by a single control parameter (codimension-one bifurcation) the saddle-node (fold) bifurcation is the generic local bifurcation.

[7] In ocean general circulation models [*Rahmstorf*, 1995], in ice-sheet dynamics [*Schoof*, 2007] and in an atmosphere sea-ice slab-ocean model [*Langen and Alexeev*, 2004] multiple steady states as a function of a single control parameter are seen.

[8] That the extremely complex dynamics of the climate system can exhibit bifurcations, which are usually associated with low order non-linear dynamical systems, is remarkable. It can be understood in terms of separation of time scales, where the fast climate variations, governed by the Navier-Stokes equation, thermodynamics and so on are effectively de-correlated and acts as a random noise forcing on the slow climate variables [*Hasselmann*, 1976]. This description is, even in the linear approximation, very successful in explaining the red noise spectra observed in climate records.

[9] The underlying assumption of the existence of different distinct climate equilibria is that there exist a set of slowly varying variables x_i, $i = 1, \ldots, N$, where N is not necessarily a small number, such that the climate dynamics can be described by the set of governing equations $\dot{x}_i = F_i(x_1, \ldots, x_N) + \sigma_i\eta_i$. At the equilibrium state (per definition) we have $\dot{x}_i = 0$ for all i, and we can linearize; $\dot{x}_i = J_{ij}x_j + \sigma\eta_i$, where now x_i represents the deviation from the equilibrium value and J_{ij} is the Jacobian of F_i at the equilibrium. Sufficiently close to a bifurcation point the most unstable direction in phase space dominates the behavior of the system. This direction is determined by the eigenvector of the Jacobian corresponding to the eigenvalue for which the eigenvalue becomes zero real valued, as a function of a control parameter, at the bifurcation point. This is why the system can be represented by a single effective variable x [*Ditlevsen*, 2004]. The dynamics of the effective variable x are governed by the Langevin equation,

$$\dot{x} = -\partial_x U_\mu(x) + \sigma\eta, \tag{1}$$

where $U_\mu(x)$ is a potential, η is a white noise and σ is the intensity of the noise. For small noise intensity we may expand around the relevant minimum, x_0, (taken to be 0 for convenience): $U_\mu(x) = U_\mu(0) + \alpha_\mu x^2/2$. Then equation (1) becomes the linear Ornstein-Uhlenbeck process: $\dot{x} = -\alpha_\mu x + \sigma\eta$. The fluctuation-dissipation theorem then gives $\langle x^2\rangle = \sigma^2/(2\alpha_\mu)$. If a bifurcation point is approached typically a local maximum and a local minimum in the potential $U_\mu(x)$ merge, such that the local minimum disappears and the system jumps into another stable state (local minimum of $U_{\mu(x)}$). In this process we have $\alpha_\mu \to 0$ for $\mu \to \mu_0$. Thus the variance grows as the bifurcation point is approached. Likewise the autocorrelation, given as $c(t) = \exp(-\alpha_\mu|t|)$, will have an increasing correlation time $T = 1/\alpha_\mu$ as the bifurcation point is approached. This is the phenomenon of critical slow down. The ratio $\langle x^2\rangle/T = \sigma^2/2$ is a constant, thus if an increased autocorrelation is seen in a data series without increase in the variance, this cannot be seen as a sign of a forthcoming tipping point.

[10] In order to investigate the significance in detection of a tipping point from a data series, two simulations of the Langevin equation (1) with a double well potential $U_\mu(x) = x^4/4 - x^2/2 - \mu x$ are performed. In the first the control parameter $\mu(t)$ is changing linearly with time, such that the bifurcation point μ_0 ($= -2\sqrt{3}/9$) is reached at time $t = 900$ time units; $\mu(t) = \mu_0 * t/900$. A realization is shown in Figure 2a with $\sigma = 0.1$. Note that the system jumps at some time prior to the bifurcation, since the potential barrier becomes small in comparison to the intensity of the noise.

[11] In the other case (Figure 2b) the parameter $\mu = 0$ is kept constant. This simulation is run for a long time, with $\sigma = 0.25$, until a purely noise induced jump from the one steady state to the other occurs. The time is then reset to zero 900 time units prior to the jump. In the first scenario variance and autocorrelation time increase prior to the jump, while in the second scenario this is not the case. The red curves show the steady states as a function of time. We want to be able to distinguish between these two scenarios prior to the jump. Especially, in the first case we want to be able to distinguish between a true warning and a false alarm due to a coincidental fluctuation in variance $\langle x^2\rangle$ and autocorrelation $\langle x(t)x(t+1)\rangle/\langle x^2\rangle$. This is done in the first column for the variance (Figure 2c) and autocorrelation (Figure 2e) calculated within a running window of 100 time units, as indicated by the black bar. The scenario is compared to the steady state scenario, corresponding to $\mu = 0$ and $\sigma = 0.1$, where no jumps occur (crosses).

[12] The analytic values for the two scenarios are plotted as blue curves (a constant in the no-jump scenario). The gray bands are the two-sigma (2Σ) levels for the calculated variance and autocorrelation within the given window size. This uncertainty (denoted Σ, not to be confused with σ) is given as $\Sigma = \Sigma_0/\sqrt{n}$, where n is the number of independent measurements in the window and Σ_0 is a constant determined by simulation. Obviously, in the second scenario with $\sigma = 0.25$ in the right column, there will be no early warning. In the first column we see that the detection of increased

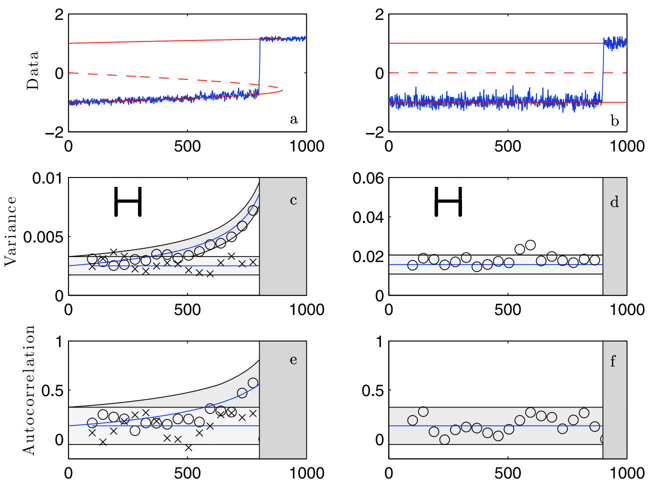

Figure 2. (top) Realizations of stochastic motion in a double-well potential. In the first case (a) a bifurcation point is approached in time, where the residing stable state disappears and the system jumps into the other remaining stable state. The jump happens some short time before the true bifurcation, where the potential barrier is small in comparison to the noise intensity. (b) The potential is static, there is no bifurcation and the jump from the lower - to the upper stable state is purely noise driven. The red curves shows the steady states as functions of time. (c and d) Open circles shows the variance calculated (for situations of Figures 2a and 2b, respectively) within a running window of 100 time units, indicated by the black bars. In (c) the crosses are calculated from a control run (not shown), with the potential frozen in the initial $t = 0$ situation. For the control run there is no jump. The curves in the grey bands are the analytic values and the grey bands are the 2-sigma confidence levels, when calculated within the given window. (e and f) Similar calculations of the 1 time unit autocorrelation. Note that for the autocorrelation the 2-sigma band is much wider than for the variance.

variance is more significant than the detection of increased autocorrelation. The window size chosen is a trade-off between a short window with too large two-sigma bands and a too long window for which the bias from the signal being non-stationary within the window becomes too large (see auxiliary material).[1]

[13] The most pronounced abrupt global climate jumps that have been observed are the Dansgaard-Oeschger climate events in the last glacial period. Note that these are separated in time scale from the terminations of the glacial periods themselves. These are also climate transitions, induced by the orbital forcing, between different climatic states. On much longer timescales even the transition between the Eocene greenhouse climate and the present Pleistocene ice-house climate might be such a transition.

[14] Here high temporal resolution isotope records of the transitions from the NGRIP ice core [*North GRIP Members*, 2004] are analyzed. The temporal resolution is such that it should be possible to detect if the jumps were preceded by early warnings.

[1]Auxiliary materials are available in the HTML. doi:10.1029/2010GL044486.

[15] Figure 3 shows 17 DO events after 60 kyr B2k, dated by annual layer counting [*Svensson et al.*, 2008], aligned such that the transitions all begin at $t = 900$ years. The blue curves (Figure 3, top) are 100 years smoothed records. The red curve is the approximately 1 year resolution of the (randomly chosen) DO4. Figure 3 (middle) is the running variance calculated from each of the high-resolution transitions. The length of the window is indicated by the black bar, the red line is the mean. Figure 3 (bottom) is the corresponding autocorrelation. Both are calculated in exactly the same way as in the model data shown in Figure 2.

[16] None of the transitions show any (significant) sign of increased variance and autocorrelation prior to the jumps. This strongly suggest that the jumps are not caused by the approach to a bifurcation point governed by some external control parameter (changing solar forcing has been suggested [*Braun et al.*, 2005]). Smoothing of the paleoclimatic record due to diffusion, limited temporal resolution etc. will in general lead to an increase in the auto-correlation in the signal. However, since these processes do not depend on the transitions there is no bias toward an increase or the opposite at the transitions. The finding suggests that internal noise (short time scale fluctuations) is the driver for these climate jumps, which implies that they will not be predictable until they actually are about to happen. This is consistent with the

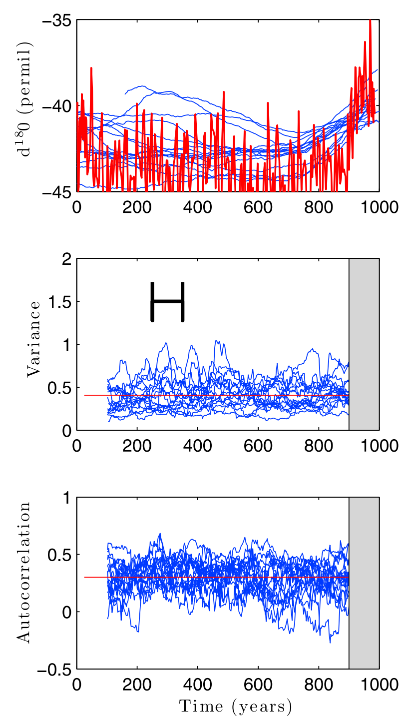

Figure 3. High resolution isotope data from the NGRIP icecore. (top) Twenty-five DO events, including the last termination, aligned such that the transitions all begins at $t = 500$ years. The blue curves are 100 years smoothed records, the red curve is the approximately 5 year resolution of the (randomly chosen) DO4. (middle) The running variance calculated from each of the high resolution transitions. The length of the window is indicated by the black bar, the red line is the mean. (bottom) The corresponding autocorrelation. Both are calculated in exactly the same way as in the model data shown in Figure 2. There are no signs of a bifurcation, indicating that the jumps are noise induced.

finding that the observed waiting time distribution between consecutive events is well fitted by an exponential, corresponding to a memory-less Poisson process [*Ditlevsen et al.*, 2007].

[17] A scenario that could be speculated for future abrupt climate jumps would be the case that the intensity of fast fluctuations increase, such that σ in equation (1) increase, while the drift term is unchanged (μ = constant), this would lead to a noise induced transition with an increased variance, but a constant autocorrelation. Note also that cases reported by *Dakos et al.* [2008], where apparently an increasing autocorrelation not accompanied by an increased variance would, if they corresponded to a bifurcation, require a fine tuned dependency between the intensity of the noise and the linear drift: $\sigma^2 \sim \alpha_\mu$.

[18] In conclusion, the early warning of climate changes or structural change in any dynamical system driven through a bifurcation, can only be obtained if increase in both variance and autocorrelation is observed. Conclusions drawn based solely on one of the signals and not the other are invalid. Furthermore, detecting increased autocorrelation, or critical slow down, with statistical significance is difficult. For the DO climate transitions, increased variance and autocorrelation are not observed. These shifts are thus noise induced with very limited predictability, and early detection of them in the future might be wishful thinking.

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