Reply

Markus Jochum

National Center for Atmospheric Research, Boulder, Colorado*

Paola Malanotte-Rizzoli

Massachusetts Institute of Technology, Cambridge, Massachusetts

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ABSTRACT

A previous study on the generation of equatorial subsurface countercurrents is revisited to clarify some details of the assumptions that are needed to derive the momentum budget. The opportunity is also used to put the study into the context of other previous studies that use quasigeostrophic theory to generalize the transformed Eulerian mean equations.

1. Reply

In Jochum and Malanotte-Rizzoli (2004, hereinafter JM) we showed that in a numerical model of the Atlantic Ocean, the South Equatorial Undercurrent (SEUC, the name for the Tsuchiya jet in the South Atlantic) is driven by the Eliassen–Palm (EP) flux convergence of the Tropical Instability Waves (TIWs). Four pieces of evidence were presented.

1) Although the SEUC is in geostrophic balance, it vanishes after the TIWs are switched off.
2) The decay time of the SEUC matches the predictions from our theory.
3) The spatial structure of the SEUC (meridional distance to the equator and depth–longitude relation) mirrors the structure of the TIWs.
4) In the core of the SEUC, EP flux convergence is balanced by viscosity.

In their comment, Marin et al. (2005, hereinafter MHS) claim that, to arrive at evidence piece 4, a residual meridional streamfunction has to exist and that the zonal pressure gradient cannot be neglected. However, the existence of a residual meridional streamfunction is *due to* the particular formulation of TEM and *not a condition* for it. It is the effect, rather than the cause. Jochum and Malanotte-Rizzoli (2004) discuss in detail that their derivation for piece 4 is strictly valid only in the core of the SEUC, at only one point at every longitude. There cannot possibly be a residual meridional streamfunction. Concerning the zonal pressure gradient, it is stated in JM that the TEM framework is modified. Jochum and Malanotte-Rizzoli (2004) use quasigeostrophic (QG) theory to arrive at their results: the zonal pressure gradient does not enter the analysis because it is balanced by the geostrophic meridional flow. Therefore, Eq. (8) of MHS is not the equation that is discussed in JM. Rather, JM separate the zero-order balance (geostrophy) from the first-order balance [QG; see their Eq. (12)]. There is not much more to say to MHS's comment because this misunderstanding about the basic assumptions in JM lies at the heart of their comment. It is unfortunate that JM did not discuss these fundamental assumptions in more detail.

Apart from this reply to the objections of MHS, we will use this opportunity to put our work in the context

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Corresponding author address: Markus Jochum, NCAR, Rm. 415, P.O. Box 3000, Boulder, CO 80307.
E-mail: markus@ucar.edu

of previous studies that combine the ideas of TEM with QG. It should have been done in JM, but we became aware of them only recently. We will also clarify and correct two details in the derivation of evidence piece 4.

2. Revisiting the original paper

JM use QG [see their Eq. (7)] and then apply ideas that have their historical roots in TEM (Eliassen and Palm 1961; Andrews and McIntyre 1976). Whereas TEM is based on the zonal mean of the equations of motion, JM is based on the time mean of the QG equations. On these already simplified equations, a technique similar to that leading to the TEM is then used. At the time of the publication of JM we were not aware of a part of the oceanographic literature that is based on exactly these very assumptions. However, an excellent discussion of these attempts is provided by Cronin (1996), and we will provide here a brief summary.

Because zonal averages are not suitable for oceanographic purposes, there have been efforts in formulating an eddy forcing theory for time-mean flows analogous to TEM (Hoskins et al. 1983; Plumb 1986; Cronin 1996). The basic physics that underlies these theories (and TEM) is that eddies can accelerate or generate a mean flow directly through advection of momentum or indirectly through advection of layer thickness. The latter case (dominant in JM) causes a steepening of the isotherms, which accelerates the flow via the thermal wind relation. A direct comparison of these two effects is not straightforward, but for the *zonal mean* case TEM provides a technique to combine both processes into the momentum equation. With the QG approximation, a similar technique can also be applied to the *time mean* fields in the ocean. It is key, however, that the eddy fluxes can be separated into a divergence-free and a rotation-free component [A. Plumb 2004, personal communication; see also the development from Eqs. (27) to (37) in Cronin (1996)].This separation is not unique and has to be argued for on a case-by-case basis (Plumb 1983; Marshall and Shutts 1981; Cronin 1996; Fox-Kemper et al. 2003). These studies provide important insights into eddy dynamics, but the exact quantification of the eddy forcing onto the mean flow remains elusive. Jochum and Malanotte-Rizzoli (2004) unknowingly sidestepped these problems by taking advantage of the particular structure of the SEUC: In its core, under the QG approximation, the equations of motion collapse to two dimensions and the ambiguity caused by the separation of the eddy fluxes does not appear. However, the referee of this reply pointed out that the assumption that the continuity equation can be reduced two dimensions cannot be derived from first principles.

We will use the opportunity here to clarify and correct two steps in the derivation of evidence piece 4 that have not been addressed properly in JM. The key in JM (as in TEM) is that the continuity equation can be reduced to two dimensions; this has to be modified:

$$u_x + v_y + w_z = 0. \tag{1}$$

Under QG assumptions we can separate between geostrophic and ageostrophic components:

$$\mathbf{u} = \mathbf{u}^g + \varepsilon \mathbf{u}^a \ (\varepsilon \ll 1); \tag{2}$$

thus

$$u_x^g + v_y^g = 0 \tag{3}$$

and

$$u_x^a + v_y^a + w_z = 0. \tag{4}$$

Without further justification, JM assumed u_x^a to be zero. Experience with equatorial dynamics suggests that $L_x \gg L_y$, but to our knowledge there is nothing that would prohibit $u^a \gg v^a$, thereby invalidating JM's assumption. In principle it would be straightforward to compute the ageostrophic velocities from the model output and verify it. As pointed out in JM, however, under the rigid-lid assumption the surface pressure was not saved and the exact values of u_a and v_a are lost. The first author is currently setting up a simulation of the tropical Pacific Ocean with a free surface to address this issue.

The fact that we cannot demonstrate that $u_x^a \ll v_y^a$ is a weak link in JM's derivation of the momentum budget. However, because the momentum budget in JM is almost closed, we will argue that the error we made by assuming $u_x^a = 0$ is negligible. Thus, we will proceed with

$$g(x, y, z) + v_y^a + w_z = 0 \tag{5}$$

[$\delta G/\delta y = g$ and $O(g) = O(v_y^a)$] instead of the two-dimensional continuity equation used in JM [their Eq. (3)].

The step from Eqs. (9) to (10) in JM also requires some explanation. The temperature balance for $\overline{(u'T')_x} \ll \overline{(v'T')_y}$ is

$$\overline{w}\,\overline{T_z} = -\overline{(v'T')_y} - \overline{v}\,\overline{T_y} - \overline{u}\,\overline{T_x} + \overline{H}, \tag{6}$$

H being the diabatic heating—here the effect of diffusion. It is possible to rewrite the vertical velocity, similar to TEM:

$$w^* = \overline{w} - \frac{1}{T_z}\overline{(v'T')_y}. \tag{7}$$

From the continuity equation above it is now possible to derive an expression for v^a and combine the eddy effects on momentum and buoyancy (in JM reduced to temperature) into one equation. The definition of w^* does not require any physical reasoning—it is pure algebra. However, under the assumption that the horizontal flow is along isotherms and the zonal derivative of the ageostrophic zonal flow as well as diffusion is negligible, the residual velocities v^* and w^* are zero and eddies have no effect on the mean flow (Eliassen and Palm 1961). Thus, the choice above is certainly not the only one, but it is a very helpful one because it allows us to quantify the effect of eddies on the mean flow. It is obvious that the flow is never strictly along isotherms and there is always diffusion, and so there will be a residual flow. If the residual flow is a dominant contributor to the momentum balance, one may have to reassess the assumptions above.

After accounting for the uncertainties in JM's assumptions, the final momentum budget is

$$f(v^* - G) = k^z\overline{u_{zz}} + k^y\overline{u_{yy}} - \overline{(u'u')_x} - \overline{(v'u')_y} + f\left[\frac{1}{T_z}\overline{(v'T')}\right]_z. \quad (8)$$

Here $k^z u_{zz}$ and $k^y u_{yy}$ are the vertical and horizontal dissipation, respectively. The difference from the final budget in JM is the term G on the left-hand side. The residual shown in Fig. 13 of JM is then due not only to the residual velocity but also to the zonal divergence of the ageostrophic zonal flow. Along the core of the SEUC the sum of these two components is smaller than any of the terms on the right-hand side (JM), which, according to JM, are important for the dynamics of the SEUC. There are two possible interpretations of this result: they a posteriori justify the simplifications of JM, or the errors introduced by JM's simplifications are large but happen to cancel each other.

3. Discussion and summary

Despite this ambiguity in the derivation of the momentum budget we still maintain that eddy fluxes from TIWs are one forcing mechanism for the SEUC, because the momentum budget is only one out of four pieces of evidence. Evidence pieces 1–3 by themselves show that the TIWs force the SEUC; the momentum budget described in piece 4 merely suggests that the eddy thickness flux is more important than the eddy momentum flux. Jochum and Malanotte-Rizzoli (2004) provide a clear physical process that explains, unlike any other current theory, the latitude of the SEUC core and why the SEUC core on its way east rises in depth *and* across isopycnals. Furthermore, it can immediately be tested by observations because it is a local theory.

Hua et al. (2003) reject this idea and claim that in their own model study they find no evidence that lateral eddies have any effect on the SEUC. However, in the core of their simulated SEUC the eddy momentum fluxes (which, at least in JM, are much less important than the thickness fluxes) are larger than the mean advection of momentum (cf. their Figs. 14c,d). This is in stark contrast to their claims.

In our opinion the comment of MHS showed that the basic assumptions underlying JM were not clearly discussed in JM; the comment of MHS does not, however, invalidate the approach of JM. Like many authors before them, JM used the QG approximation to manipulate the resulting equations in a way similar but not identical to TEM. The difference between the approach of JM and previous authors is that JM assume that the particular structure of the SEUC allows for a reduction of the QG equations to two dimensions, thereby avoiding the ambiguity associated with the more general three-dimensional approach of earlier studies. However, during the review of the present paper the referee pointed out that a key assumption in JM ($u_x^a \ll v_y^a$), which appeared obvious to JM, cannot be justified from first principles, and JM do not have the model output to verify it.

It is shown here that the smallness of the residual of the momentum budget could be interpreted as a posteriori justification of JM. However, this is not the only possible interpretation, and the present authors have to analyze the structure of the ageostrophic velocity fields in a new model experiment before a definite conclusion can be made.

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