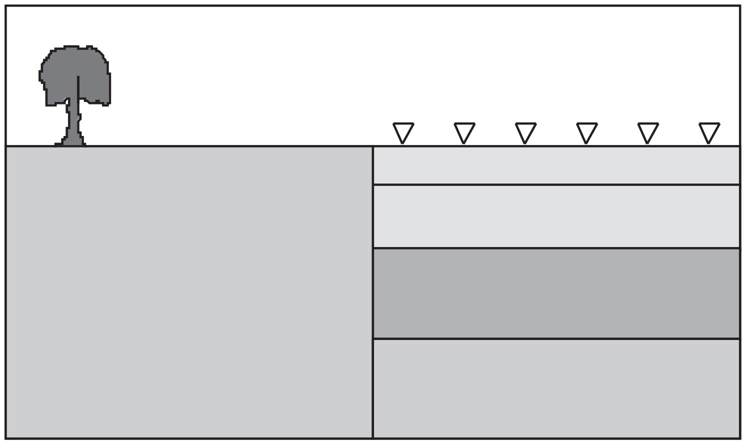
Computer Assignment – Part 3

Monte Carlo analysis of a nonlinear inverse problem



Again, using the same data as in Computer Assignment 1, we wish to estimate $\Delta\rho(z)$ from observations of the horizontal gravity gradient along the x-axis. In this assignment we shall use the same parameterization of the problem as in Computer Assignment 2, but this time we shall use the the Metropolis Algorithm to sample solutions to the problem. The distribution of solutions is the so-called *a posteriori* probability density $\sigma(\mathbf{m})) = \rho(\mathbf{m})L(\mathbf{m})$. In this exercise we will assume that the likelihood function $L(\mathbf{m})$ has the following form:

$$L(\mathbf{m}) = K_1 \exp\left(-\frac{\|\mathbf{d}_{obs} - \mathbf{g}(\mathbf{m})\|^2}{2\sigma^2}\right) , \quad (1)$$

where K_1 is a normalization constant, and σ is the standard deviation of the noise. We assume that $\sigma = 1.0 \cdot 10^{-9} s^{-2}$ (shown as "error bars" in Figure 2). Furthermore, we shall assume that the prior probability density is constant:

$$\rho(\mathbf{m}) = K_2 \quad (2)$$

indicating that no models are preferred over others.

- Find, using the Metropolis Algorithm, a large number of models $\mathbf{m}^{(n)}$, $n = 1 \ldots N$, distributed according to the *a posteriori* distribution.

If we define the *energy* (or *misfit*)

$$E(\mathbf{m}) = -\ln(\sigma(\mathbf{m})) ,$$

each iteration of the Metropolis algorithm runs as follows:

1. Perturb the current model $\mathbf{m}_k$ (in the first iteration, choose a random model) by choosing a random component of $\mathbf{m}_k$ and adding a random number from the interval $[-S, S]$, where S is a maximum step length.

2. The perturbed model $\mathbf{m}_k^{pert}$ is only accepted as the next model ($\mathbf{m}_{k+1}$) with the probability

$$p_{accept} = \begin{cases} \exp(-(E(\mathbf{m}_k^{pert}) - E(\mathbf{m}_k)) & \text{for } E(\mathbf{m}_k^{pert}) > E(\mathbf{m}_k) \\ 1 & \text{otherwise.} \end{cases} \tag{3}$$

   If $\mathbf{m}_k^{pert}$ is accepted, put $\mathbf{m}_{k+1} = \mathbf{m}_k^{pert}$. If, on the other hand, $\mathbf{m}_k^{pert}$ is rejected, put $\mathbf{m}_{k+1} = \mathbf{m}_k$.

3. Repeat (1) and (2) with $\mathbf{m}_{k+1}$ as the current model.