Inversion of seismic reflection data

In this problem we start with a calculation of artificial (synthetic) seismic data from a known seismic waveform (a *wavelet*), and known earth properties (a *reflectivity*). The file "wavelet.txt" contains the wavelet $w(t)$, as a function of time t (see Figure 1), generated by a dynamite explosion at the earth's surface. Here we will only consider seismic rays that travel vertically downwords, and are reflected vertically back from layer interfaces to the surface, where the seismogram is recorded. This is the case when the earth is horizontally stratified.

The file "reflectivity.txt" (Figure 2) contains the reflectivity function $r(t)$ measured in a well next to the explosion point. $r(t)$ is defined as the function whose value at time t is equal to the seismic reflection coefficient at depth $z = \frac{1}{2}vt$, where v is the average wave speed above z.

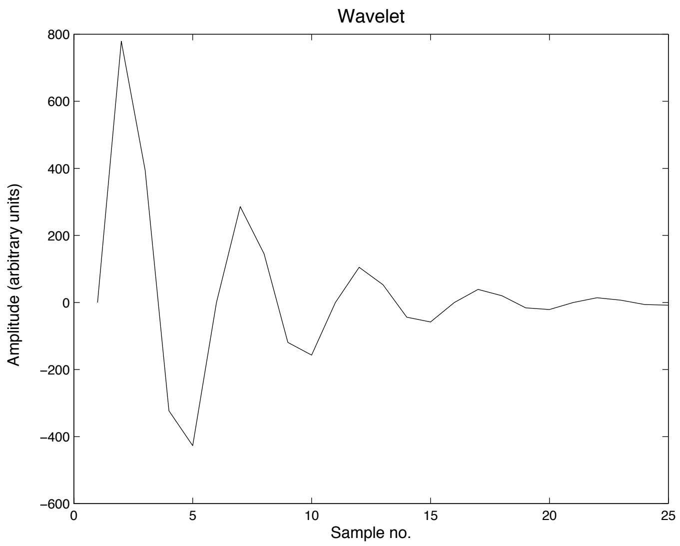

Figure 1: The seismic pulse (wavelet) generated by an explosion.

According to the so-called "convolutional model", the seismogram $s(t)$ generated by a seismic energy source, located at the well site (at $z = 0$), and recorded in the same point, is theoretically given by

$$s(t) = r(t) * w(t) \equiv \int_{-\infty}^{\infty} r(\tau)w(t-\tau)d\tau.$$

where "$*$" denotes convolution (as defined by the integral). The above formula can be discretized through

$$s_i = \sum_{j=1}^{i} r_j w_{i-j+1}$$

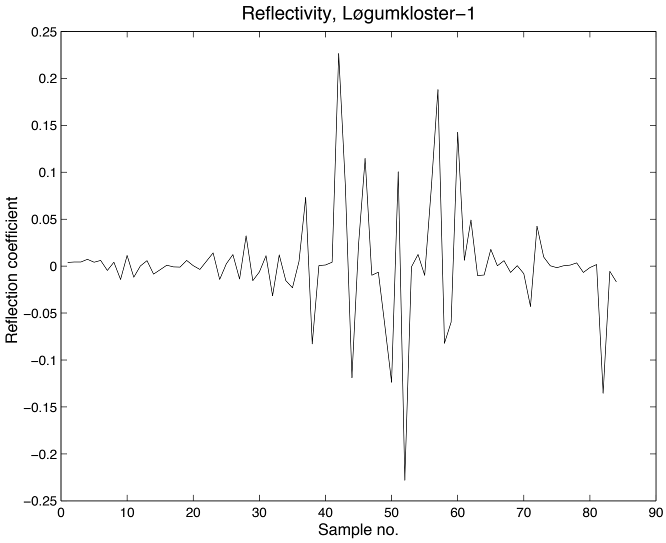

Figure 2: The reflectivity measured in the well.

where s_i, w_i, and r_i are the i'th samples of the discrete seismogram, wavelet and reflectivity, respectively.

The forward problem: Generation of synthetic data

1. Compute a discrete, noise-free seismogram $\mathbf{s}^{pure} = \left\{ s_i^{pure} \right\}$ from the discrete wavelet $\mathbf{w} = \{w_i\}$ and the discrete reflectivity $\mathbf{r} = \{r_i\}$, found in the files "wavelet.txt" and "reflectivity.txt".

2. Simulate observed data $\mathbf{s}^{obs}$ by adding noise $\mathbf{n}$ to the "pure" data

$$\mathbf{s}^{obs} = \mathbf{s}^{pure} + \mathbf{n}$$

   where $\mathbf{n}$ has independent, normal distributed components with zero mean, and where $\|\mathbf{n}\| = 0.20 \, \|\mathbf{s}^{pure}\|$. Plot $\mathbf{s}^{obs}$ and $\mathbf{n}$.

The inverse problem

3. Formulate the discrete inverse problem as a matrix equation

$$\mathbf{d} = \mathbf{G}\mathbf{m} \, . \tag{1}$$

4. Is the problem well-determined, purely under-determined, purely over-determined or mixed-determined?

5. A singular value decomposition of $\mathbf{G}$ can be expressed

$$\mathbf{G} = \mathbf{U}\mathbf{D}\mathbf{V}^T \ . \tag{2}$$

where $\mathbf{U}$ and $\mathbf{V}$ are orthogonal matrices, and $\mathbf{D}$ is a diagonal matrix with the singular values $\lambda_1 \geq \lambda_2 \geq \cdots \geq 0$ in its diagonal. Find a solution to the problem using *Truncated Singular Value Decomposition* (TSVD), where we use an inverse operator of the form

$$\mathbf{H} = \mathbf{V}_p \mathbf{D}_p{}^{-1} \mathbf{U}_p{}^T \ . \tag{3}$$

Here, the matrices $\mathbf{V}_p$ and $\mathbf{U}_p$ consist of the first p columns of $\mathbf{V}$ and $\mathbf{U}$, respectively, and $\mathbf{D}_p$ is the $p \times p$ diagonal matrix having the first p singular values $\lambda_1, \lambda_2, \ldots, \lambda_p$ in its diagonal. Choose p such that the solution is physically acceptable, gives the best possible resolution of the model, and, at the same time, "barely fit the data within the noise".

6. Investigate the the model resolution. Plot for instance how a delta function (zero everywhere, except in a single point) in the true model is reconstructed in the estimated model.

7. Calculate the uncertainty (dispersion/standard deviation) for the estimated model.