Observation of α-stable noise induced millennial climate changes from an ice-core record

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Abstract. The last glacial period showed millennium scale climatic shifts between two different stable climate states. The state of thermohaline ocean circulation probably governs the climate, and the triggering mechanism for climate changes is random fluctuations of the atmospheric forcing on the ocean circulation. The high temporal resolution paleoclimatic data from ice-cores are consistent with this picture and a bi-stable climate pseudo-potential can be derived. It is found that the fast time scale noise forcing the climate contains a component with an α-stable distribution. As a consequence the abrupt climatic changes observed could be triggered by single extreme events. These events are related to ocean-atmosphere dynamics on annual or shorter time scales and could indicate a fundamental limitation in predictability of climate changes.

Paleoclimatic records from ice-cores [*Dansgaard et al.*, 1993] show that the climate of the last glacial period experienced rapid transitions between two climatic states, the cold glacial periods and the warmer interstadials (Dansgaard-Oeschger events). Deep sea sediment-cores [*Bond et al.*, 1993] and coral records [*Beck et al.*, 1997] indicate that the ocean circulation is a key player in these climatic oscillations [*Broecker et al.*, 1985]. Ocean circulation models, from the most simple Stommel type [*Cessi*, 1994] to the complex circulation models [*Rahmstorf*, 1995] show that different flow states can exist as meta-stable climatic states. The changes in the thermohaline circulation of the Atlantic has probably been such that North Atlantic Deep Water (NADW) is produced in the warm periods and North Atlantic Intermediate Water in the cold periods [*Duplessy et al.*, 1988; *Stocker and Wright*, 1996].

The key question is then whether the switching between two stable climatic states of the oceanic flow can be internally triggered by the random forcing within the ocean-atmosphere system [*Broecker*, 1997].

This stochastic climate dynamics is described by a Langevin equation [*Cessi*, 1994], $dy = -(dU/dy)dt + dN$. The variable y represents the climatic state, which could be associated with the pole ward heat transport or the NADW [*Rahmstorf*, 1995]. The first term on the right hand side represents the dynamics of the ocean circulation where U is the climate pseudo-potential. The potential describes the bi - or multi state character of the climate system. The second term on the right hand side is a noise term representing the atmospheric forcing, through the wind stress, heating and freshwater transport, on the climate state.

Taking the ice-core record to be a climatic proxy resulting from the climate dynamics described through the Langevin equation, the aim of this work is to confirm that this is a consistent description and from the analysis to observe the structure of the noise driving the system. This provides strong constraints on the types of possible models of the underlying triggering mechanisms for the observed climatic shifts.

The calcium signal from the GRIP ice-core is the highest temporal resolution glacial climate record which exists [*Fuhrer et al.*, 1993]. The logarithm of the calcium signal is (negatively) correlated with the $\delta^{18}O$ temperature proxy with a correlation coefficient of 0.8 [*Yiou et al.*, 1997], thus we use the logarithm of calcium as a climate proxy since it is related to dust in the ice and it is therefore not diffusing in the ice like the $\delta^{18}O$ signal. The temporal resolution of log(Ca) is about annual from 11 kyr to 91 kyr B.P. (80,000 data-points). This is an order of magnitude higher than that of $\delta^{18}O$. The calcium signal from the GRIP ice-core is shown in figure 1 a. The typical waiting time for jumping from one state to the other is between 1000 and 2000 years. The probability distribution for the waiting times between the beginning of the glacial - and the beginning of the following interstadial states is shown in figure 2. The straight line is an exponential distribution with mean waiting time of 1400 years. This is expressed as, $P(T > t) = \exp(-t/\tau)$, where $\tau = 1400$ years is the mean waiting time. The data thus clearly shows that the shifting is not, as speculated before, a periodic oscillation. On the contrary, this observation

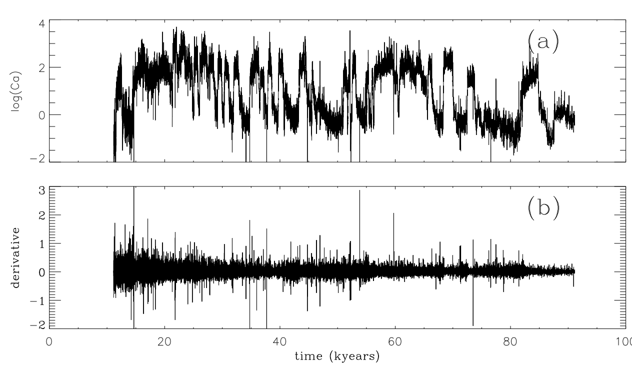

Figure 1. (a) The logarithm of the calcium concentration as a function of time (BP) in the GRIP ice-core. The dating of this upper part of the record is rather precise. The temporal resolution is about 1 year, much better than the $\delta^{18}O$ record since the dust does not diffuse in the ice. The signal is a proxy for the climatic state. (b) The derivative of the signal in (a). This approximately stationary signal is strongly intermittent.

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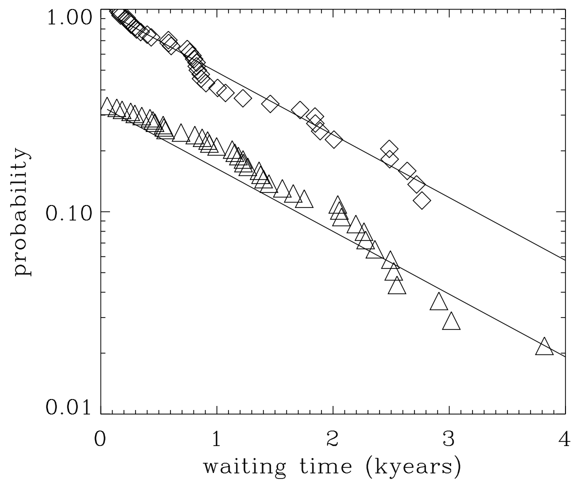

Figure 2. The jumping between the glacial and interstadial states is well described as a Poisson process. The waiting times are defined as the times between consecutive first up crossings through the level log(Ca)=2 (glacial state) and first down crossings through the level log(Ca) = -0.6 (interstadial state). The waiting times have an exponential distribution (diamonds). The full line is an exponential distribution with a mean waiting time of 1400 years. The data record is not long enough to determine if the waiting times for the interstadials and the glacial states are significantly different. The exponential distribution is consistent with the stochastic dynamics. The triangles, which are vertically shifted for clarity, are results from simulation (see text).

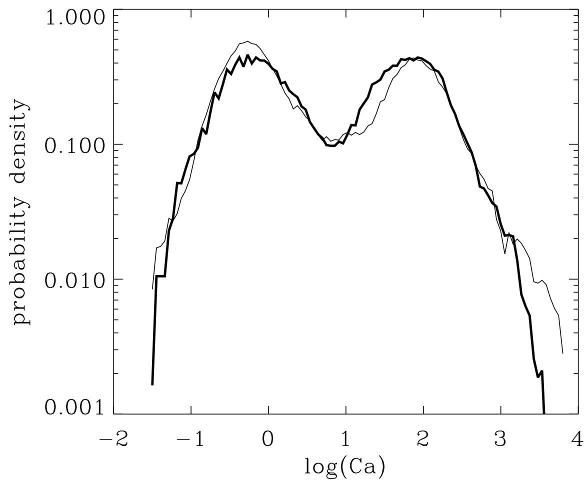

Figure 3. The probability density function (PDF) of the log(Ca) signal shows the bimodal distribution. The left maximum corresponds to the interstadial state and the right maximum corresponds to the glacial state. The thin curve is the PDF of the simulated signal (figure 6).

is consistent with the triggering mechanism for the shifting being a white noise forcing. Furthermore, the shifting is not a stochastic resonance phenomenon, since the shifting is not periodically occurring. The probability density function (PDF) of the signal, figure 3, shows a bimodal distribution with peaks corresponding to the warm interstadials and the glacial state.

From the premise of the stochastic dynamics and the data we can now uniquely determine the climate pseudo-potential, $U(y)$, and the structure of the noise term. The noise term (diffusion term) is to first order, neglecting the drift term, defined as the derivative of the signal estimated as $(y_{t+\Delta t} - y_t)/\Delta t$, shown in figure 1 (b). This signal is stationary except for a slow trend through the record which is partly due to smoothing with depth in the ice-core. The intensity of the noise is thus approximately independent of log(Ca). Note the important implication that the intensity of fluctuations in calcium is proportional to the calcium signal itself ($dx/dt = x\, d\log x/dt$) and through that to the degree of glaciation [*Ditlevsen et al.*, 1996].

The noise is approximately a white noise and has a strongly non-gaussian distribution. Figure 4 (a) shows the cumulated probability on a scale on which a gaussian distribution is a straight line (probability paper scale). Figure 4 (b) shows the two distribution tails on a log-log plot magnifying the behavior of the tails. This has in an intermediate range a power function scaling with a power of about 2.75 and an additional extreme tail. The signal can only be described by a Langevin equation with two noise terms,

$$dy = -(dU/dy)dt + \sigma_1 dx + \sigma_2 dL. \qquad (1)$$

The first noise component, $\sigma_1 dx$, is generated by an additional Langevin equation, $dx = -xdt + \sqrt{1+x^2}dB$, where x is an (unmeasured) independent variable and dB is a unit variance Brownian noise.

The stationary distribution for x is a t-distribution which fits to the observed tail distribution for the noise on y. This term describes the forcing from the atmosphere. That the noise is white means that it is uncorrelated between consecutive data points. That does not exclude that the noise is red on shorter (unmeasured) time scales. Actually the term $-xdt$ gives a correlation time of one year. It should be noted that the same signal could be scaled with a factor ρ: $dx = -\rho^2 xdt + \rho\sqrt{1+x^2}dB$ consistently with the data as long as $\rho > 1$. ρ^{-1} signifies the correlation time (which is shorter than one year). The reason that this noise term is not just a gaussian white noise term is that the intra-annual variability is strongly dependent on season, with much stronger intensity in winter than in summer, and that the inter-annual correlation leads to the red noise signal, where the 'non-gaussianity' survives the annual averaging.

The second noise term is an α-stable noise with stability index $\alpha = 1.75$ [*Samorodnitsky and Taqqu*, 1994]. The α-stable distributions, corresponding to $\alpha < 2$, have cumulative probability tails which scales as $x^{-\alpha}$ implying that only moments of order less than α exists ($\langle |x|^\beta \rangle = \infty$ for $\beta \geq \alpha$). The α-stable distributions fulfill a generalized version of the central limit theorem, namely that the distributions of sums of identically distributed random variables with cumulative distribution tails scaling as $x^{-\alpha_1}$ converges to an α-stable distribution with $\alpha = \alpha_1$. These distributions have very fat tails, meaning that the probability of extreme events is high, such that single extreme events within a period over which the variable is averaged will show up also in the distribution of the averages. The α-stable distributions were first observed in hydrological records of river flow [*Hurst*, 1951],

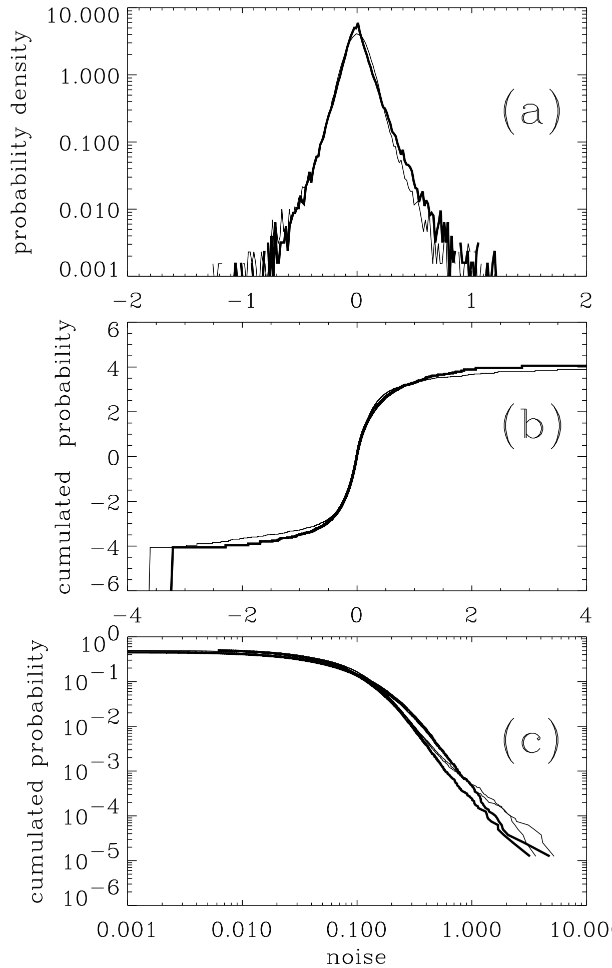

Figure 4. (a) The probability density of the noise (figure 2 b). (b) The cumulated distribution of the noise. The scale is a 'probability paper scale' where a gaussian distribution shows up as a straight line. This signal is strongly non-gaussian. (c) The two tails of (b) on a log-log plot. For the upper tail the probability of values larger than the abscissa is shown. The thin curves are from the simulation, showing that the signal is well described as containing a t-distributed noise component and an α-stable noise component.

and have later been observed in various different physical systems [*Shlesinger et al.*, 1994] such as turbulent diffusion [*Zimbardo et al.*, 1995] and vortex dynamics [*Viecelli*, 1993]. To the present there is still no full theoretical understanding of why these distributions are observed, and it has not before been noted to be of importance in climate dynamics.

A generalization of the Fokker-Planck equation for the two coupled Langevin equations with α-stable noise excitations connects the stationary density solution to the pseudo-potential $U(y)$. However, only the marginal distributions are known. For y this is the PDF for log(Ca) shown in figure 3. The pseudo-potential, shown in figure 4, is thus determined iteratively by simulation starting from a solution to the stationary one-dimensional Fokker-Planck equation using the marginal distribution.

In order to validate that the log(Ca) signal can be described by (1) a consistency check must be performed. This

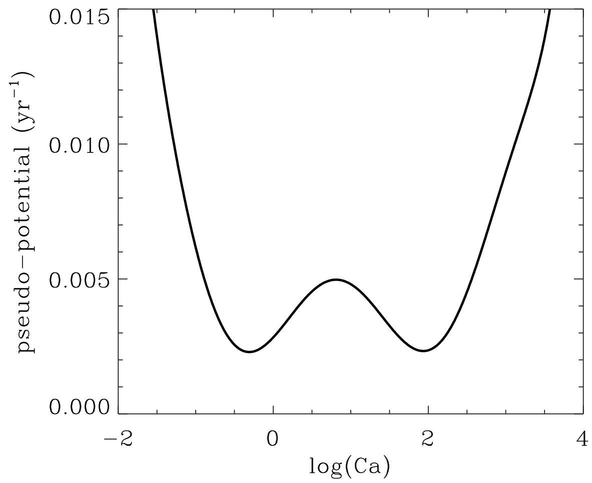

Figure 5. The climate pseudo-potential is a double-well potential with the left well representing the interstadial state and the right well representing the full glacial state. The potential is obtained from a generalized stationary Fokker-Planck equation.

is done by simulation. Using the derived pseudo-potential, figure 5, fitting σ_1 and σ_2 from the noise structure of the signal, figure 6, shows a realization of (1). This should be compared with the log(Ca) signal, figure 1 (a).

The thin lines in figures 3 and 4 are derived from the simulated signal. The stationary Fokker-Planck equation does not contain information about the time scales for jumping, figure 2, and temporal harmonic decomposition of the sample as represented by the power spectra which are compared in figure 7. These constitute independent verifications. As seen in all the figures the agreement is astonishing. Judged from different simulated realizations the two signals only deviate within the statistical uncertainty.

This is not merely an advanced curve fitting routine. If the calcium data is assumed to be generated by the dynamics described through a Langevin equation, the driving noise must be of the form described here. In order to understand the underlying climate dynamics it is important to establish the connection between this climatic proxy and the climate. It is especially important to interpret the two noise terms and connect them to the atmosphere-ocean dynamics. The noise term '$\sigma_1 dx$' is probably related to the 'normal' atmospheric fluctuations. Since the sampling is coarse on the

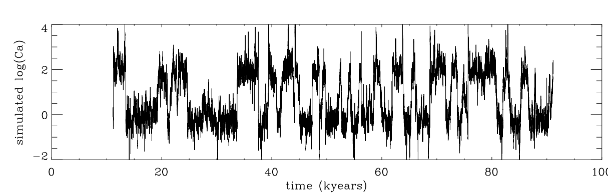

Figure 6. An artificial log(Ca) obtained from simulating a sample solution to the Langevin equation using the climate pseudo-potential, an $\alpha = 1.75$ white noise and $\sigma_1/\sigma_2 = 3$. This should be compared to figure 2 a. The two signals are statistically similar, showing that the log(Ca) signal can be generated by the stochastic dynamics.

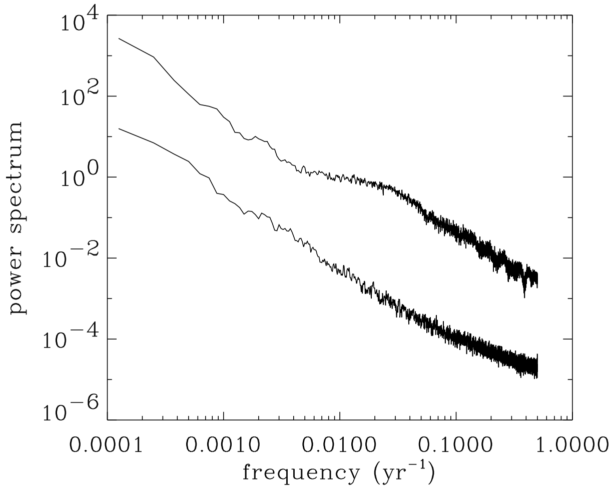

Figure 7. The temporal harmonic resolution as expressed through the power spectrum is an independent measure of the signal. The top curve is the power spectrum of log(Ca). This is a red noise spectrum without significant peaks. The bottom curve is the power spectrum of the simulated signal vertically shifted for clarity.

time scales of these fluctuations, there is is an indeterminacy in the noise structure on time scales shorter than about one year. In the model this reflects itself in the invariance of this noise term with respect to ρ. The noise term $\sigma_2 dL$ represents extreme events and calls for attention. The α-stable noises seems to occur in dynamical systems with many different time scales where the dynamics becomes strongly intermittent.

The presence of an α-stable noise component could imply that the triggering mechanisms for climatic changes are single extreme events. Such events, being on the time scale of seasons, are fundamentally unpredictable and never captured in present days numerical circulation models. All coupled general circulation models will due to smoothening and coarse resolution almost certainly show gaussian statistics. This could explain why these models have yet never succeeded in simulating shifts between climatic states.

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