A two-parameter stochastic process for Dansgaard-Oeschger events

H. Braun,[1,2] P. Ditlevsen,[1] J. Kurths,[3,4] and M. Mudelsee[5,6]

Received 7 March 2011; revised 28 April 2011; accepted 9 May 2011; published 7 September 2011.

[1] Various climatic processes are thought to evolve as rapid, shift-like events, which points at the presence of nonlinear dynamics. Time series analysis of nonlinear processes, however, is not trivial, for example, because of the difficulty in coming up with a realistic random process as a viable null hypothesis. In this methodology paper we construct a basic two-parameter process of shift-like excursions in an excitable system with a threshold. We demonstrate that this stochastic process, in comparison with a specific one-parameter process, can better reproduce main features of the waiting time histogram of abrupt glacial climate events, the Dansgaard-Oeschger events, as seen in two paleoclimatic proxy records, the North Greenland Ice core Project (NGRIP) ice core and the Sofular stalagmite δ^{18}O records. We use the two-parameter process to test some arguments that were proposed in the ongoing discussion of a possible solar role in triggering Dansgaard-Oeschger events. Using our approach, we suggest for future studies to generate time series of random events which can serve as a more plausible null hypothesis for Monte Carlo based statistical tests on the regularity of shift-like processes such as Dansgaard-Oeschger events.

Citation: Braun, H., P. Ditlevsen, J. Kurths, and M. Mudelsee (2011), A two-parameter stochastic process for Dansgaard-Oeschger events, *Paleoceanography*, *26*, PA3214, doi:10.1029/2011PA002140.

1. Introduction

[2] Various climatic processes are thought to evolve as rapid, shift-like events, for example as oscillations between different modes of operation of the ocean-atmosphere system. A prominent example are the Dansgaard-Oeschger (DO) events during glacial times (Figure 1) [*Dansgaard et al.*, 1982; *Oeschger et al.*, 1984; *Broecker et al.*, 1985]. DO events are manifested for example in ice core climate proxy records from Greenland [*Dansgaard et al.*, 1982; *Grootes et al.*, 1993; *Andersen et al.*, 2006], in deep-sea sediment proxy records from the North Atlantic region [*Bond et al.*, 1993] and in stalagmite proxy records from Eurasia [*Wang et al.*, 2001; *Spötl and Mangini*, 2002; *Fleitmann et al.*, 2009]. The events are commonly interpreted as large-amplitude, approximately 10 to 15 K [*Severinghaus and Brook*, 1999; *Lang et al.*, 1999] shift-like oscillations between two different modes of glacial climate, that is, the stadial ("cold") and interstadial ("warm") mode, respectively [*Dansgaard et al.*, 1982; *Oeschger et al.*, 1984; *Broecker et al.*, 1985]. This implies an intrinsically nonlinear dynamical scenario.

[3] So far, standard methods of linear time series analysis have predominantly been used to investigate the regularity of DO events. For example, a prominent 1470 year spectral component, which is closely related to the occurrence of DO events, was reported to exist in the GISP2 ice core δ^{18}O record [*Grootes and Stuiver*, 1997]. This spectral peak was reported to be statistically inconsistent with a first-order autoregressive (AR1) random process, at a significance level of 0.01 [*Schulz*, 2002]. However, an AR1 process is a linear noise-driven random process, and recent simulations with models of different complexity indicate that the power spectral density of random DO-like events could be substantially different from an AR1 random process [*Braun et al.*, 2010]. This questions the applicability of an AR1 random process for estimating the statistical significance of the reported 1470 year spectral peak of DO events and highlights the need for nonlinear random processes as a more realistic null-hypothesis for time series analysis on the regularity of DO events and, potentially, other shift-like climate anomalies.

[4] Here we connect the statistical concept of hypothesis testing [*Lehmann and Romano*, 2005; *Ditlevsen et al.*, 2007; *Mudelsee*, 2010] with the principle of parsimony, in an attempt to construct a simple but dynamically plausible noise-driven process, with a particularly small number of parameters, that is able to reproduce main features of the waiting time histogram of DO events as seen in the North Greenland Ice core Project (NGRIP) [*Svensson et al.*, 2008]

[1]Centre for Ice and Climate, Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark.
[2]Heidelberg Academy of Sciences and Humanities, Heidelberg, Germany.
[3]Institute of Physics, Humboldt University Berlin, Berlin, Germany.
[4]Potsdam Institute for Climate Impact Research, Potsdam, Germany.
[5]Climate Risk Analysis, Hannover, Germany.
[6]Alfred Wegener Institute for Polar and Marine Research, Bremerhaven, Germany.

0883-8305/11/2011PA002140

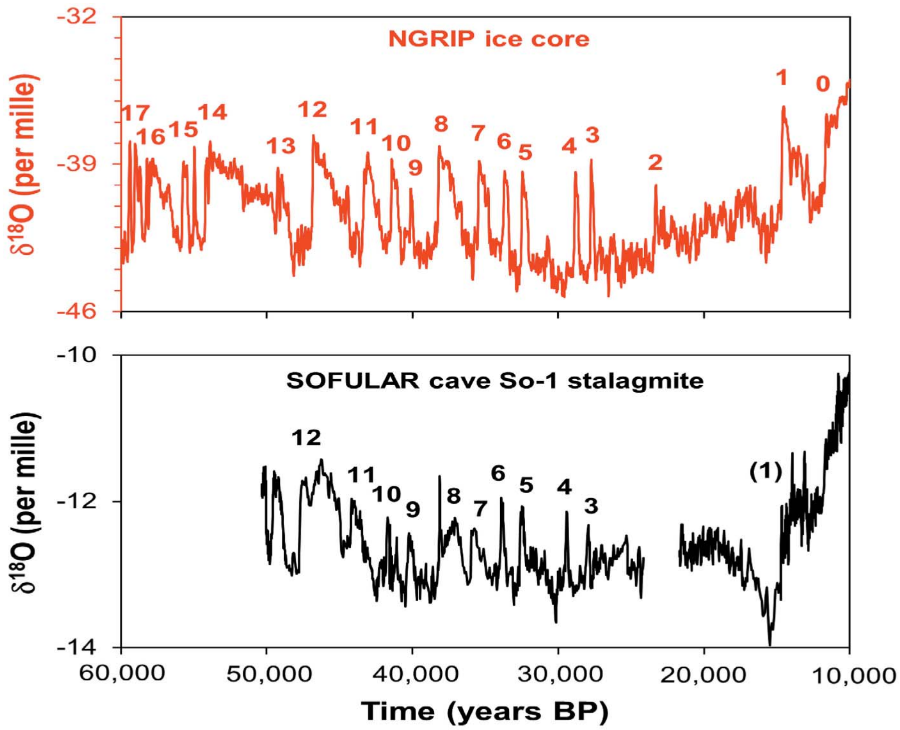

Figure 1. The most recent Dansgaard-Oeschger (DO) events as seen in two paleoclimatic proxy records. (top) The NGRIP ice core δ^{18}O record from Greenland during the time interval between 11,000 and 60,000 years before present (GICC05 time scale). (bottom) The Sofular cave So-1 stalagmite δ^{18}O record from Turkey. A five-point running mean was applied to the original NGRIP data in order to remove the highest-frequency oscillations. The numbers label the events 0–17, following standard paleoclimatic convention [*Svensson et al.*, 2008]. Here we consider the continuous part of the Sofular record, which only includes events 3–12 but not events 0–2 because the stalagmite stopped growing between about 22,000 and 24,000 years before present. Note that time evolves from the left to the right, in contrast to geological convention. Further note that the timescale of the NGRIP ice core, that is, the relation between drilling depth and the age of the ice layers, was obtained by independent counting of annual ice layers using high resolution measurements [*Andersen et al.*, 2006], whereas the timescale of the Sofular stalagmite was obtained by a combination of radiometrical ^{230}Th dating and linear interpolation [*Fleitmann et al.*, 2009].

and Sofular So-1 stalagmite [*Fleitmann et al.*, 2009] stable isotope (δ^{18}O) records. In this way, we find that the hypothesis that the interevent waiting time distribution of DO events is given by a specific one-parameter process (geometric distribution) can be statistically rejected with very high significance ($p < 0.01$) based on the continuous part of the Sofular record (containing the events 3–12) and with high significance ($p < 0.05$) based on the most recent part of the NGRIP record (containing the events 0–17). We then demonstrate that a simple two-parameter stochastic process, in which DO events are regarded as shift-like excursions in an excitable system with a threshold and a relaxation process, can better reproduce main features of the interevent waiting time histogram of DO events as recorded in the two considered paleoclimatic proxy records. We thus suggest that our proposed two-parameter stochastic process represents a promising process to describe the recurrence pattern of DO events. We further propose to use our process, which constitutes a new step in the course of attempts to describe DO generation as a stochastic process, as a more realistic dynamical process to simulate the recurrence properties of random DO events, which is needed for Monte Carlo based statistical analyses on the regularity of DO events, for example for tests on the possible solar role in triggering the events. Finally we use our process to test some arguments that were proposed in the ongoing discussion of a possible solar role in triggering DO events: We demonstrate that even in a highly simplified solar forcing scenario, a nonperiodic regularity of DO events can be expected. Our study thus indicates that more efficient measures of nonperiodic regularity are needed to distinguish between a random occurrence of DO events and a scenario in which the events are triggered at least in part by solar forcing.

2. H0: A Simple One-Parameter Random Process (Geometric Distribution)

[5] As a starting point, we construct the following very simple and – according to our interpretation – plausible nonlinear random process for the occurrence of DO events, which we regard as our null hypothesis: Let us assume that DO events represent discrete events which are triggered each time a given random input n (i.e., noise) is larger than a certain constant threshold value T. A physical process with threshold-crossing dynamics, namely the process of buoyancy deep convection, was earlier suggested as the gener-

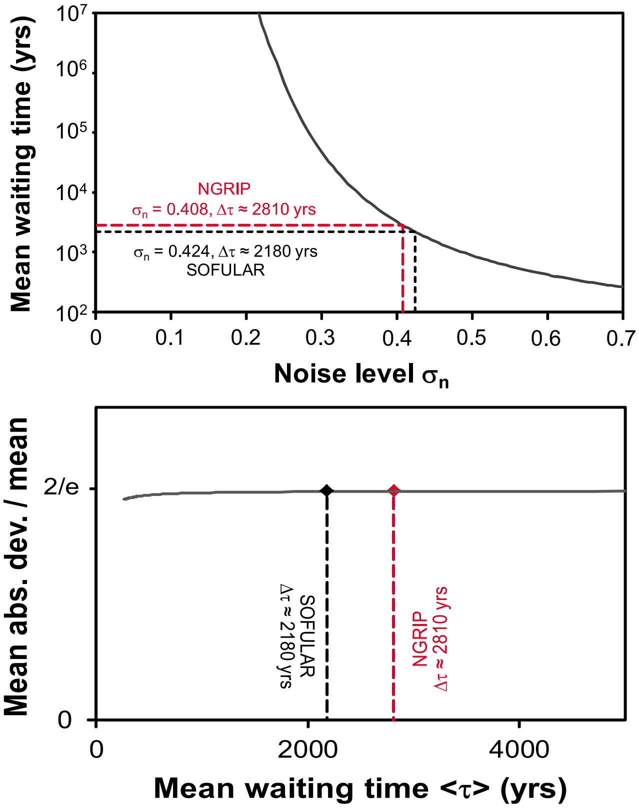

Figure 2. Waiting time properties of DO events as obtained from the one-parameter process H0 (geometric distribution). (top) The population mean interevent waiting time $\langle\tau\rangle$ as a function of the noise level (i.e., of the standard deviation σ_n of the noise). The dashed lines label the sample mean interevent waiting time $\Delta\tau$ as obtained from the NGRIP (red) and Sofular (black) proxy records, respectively (Table 1). (bottom) The ratio between the population mean absolute deviation $\langle|\tau - \langle\tau\rangle|\rangle$ (mean absolute deviation) and the population mean value $\langle\tau\rangle$ (mean) of the simulated interevent waiting time, as a function of the population mean waiting time $\langle\tau\rangle$, according to our random process H0. Again, the dashed lines label the sample mean interevent waiting time $\Delta\tau$ as obtained from the NGRIP (red) and Sofular (black) proxy records, respectively. Within the chosen range of noise levels, the ratio $\langle|\tau - \langle\tau\rangle|\rangle/\langle\tau\rangle$ is almost independent of the noise level, which is desired. The length of each simulation is 100,000,000 time steps (i.e., 2,000,000,000 years).

ating mechanism of DO events: Using an ocean-atmosphere model of intermediate complexity, it was reported that DO-like events can be generated as repeated shifts between different modes of buoyancy deep convection (i.e., deep water formation) in the northern North Atlantic [*Ganopolski and Rahmstorf*, 2001]. Results obtained with an ocean-atmosphere model of intermediate complexity further indicate that the onset of deep buoyancy convection in the northern North Atlantic under boundary conditions corresponding to the Last Glacial Maximum could be a threshold process, which occurs when the potential density of the surface water becomes large as compared with the density of the deeper ocean water [*Ganopolski and Rahmstorf*, 2001]. Moreover, the abruptness of the onset of DO events in geological climate proxy records, e.g., in ice core records [*Steffensen et al.*, 2008], is commonly regarded as additional indication for the existence of a threshold in the climate system.

[6] For simplicity, we normalize the threshold value by choosing $T = 1$. The condition for the occurrence of an event at time t is thus given by the equation

$$n(t) > 1. \tag{1}$$

Thus, in this very simple process, the waiting time distribution of the output events is given by the distribution of the waiting times between successive input anomalies with values larger than one. Let us assume $n(t) = n_i$ to be a discrete process drawn every time step "i" from a Gaussian distribution $N(0, \sigma_n)$, with zero mean and standard deviation $\sigma_n > 0$. This implies that the probability p for the occurrence of an output event at a given time step is time independent, so that the problem of determining the waiting time τ between successive events is equivalent to determining the number of Bernoulli trials that are needed to obtain a first success. This probability is given by the geometric distribution [*Johnson et al.*, 1992]:

$$P(\tau) = (1-p)^{\tau-1}p = p/(1-p)\exp(-\tau/T_0), \text{ with } T_0 = -1/\ln(1-p), \tag{2}$$

where $P(\tau)$ denotes the probability for the occurrence of a waiting time τ between successive events, p is the probability for the occurrence of an event at a given time step (i.e., for a "success") and $1 - p$ is the probability that no event occurs at a given time step ("failure"). Apart from the time step, which we chose to be 20 years since this value is the time resolution of the NGRIP $\delta^{18}O$ record and close to the average time resolution of the Sofular $\delta^{18}O$ record in the relevant time interval, this normalized random process has thus only one adjustable parameter, which is the standard deviation σ_n of the noise, or, equivalently, the probability p for a "success." This probability can be expressed as a function of σ_n in terms of the error function erf(x):

$$p(\sigma_n) = 1/2 - 1/2\,\mathrm{erf}\left(2^{-1/2}\sigma_n^{-1}\right). \tag{3}$$

We further note that the following relations hold for the mean interevent waiting time $\langle\tau\rangle$ and the population mean absolute deviation $\langle|\tau - \langle\tau\rangle|\rangle$:

$$\langle\tau\rangle = 1/p \tag{4}$$

$$\langle|\tau - \langle\tau\rangle|\rangle = 2(1-p)^{\lfloor 1/p \rfloor}\lfloor 1/p \rfloor. \tag{5}$$

In expression (5), $\lfloor 1/p \rfloor$ denotes the greatest integer smaller than or equal to $1/p$ [*Kapadia*, 1983]. The reason why we focus on the mean absolute deviation, and not, e.g., on the standard deviation, is that the waiting time between the DO events 1 and 2 is considerably larger than between the other events (Figure 1), and that the mean absolute deviation is less sensitive to the presence of such outliers than the standard deviation. We note that, so far, the geometric distribution (respectively its continuous analog, the exponential

Table 1. Timing of the Onset of the Most Recent DO Events as Inferred From the NGRIP Deep Ice Core δ^{18}O Record From Greenland (GICC05 Chronology) and the Sofular So-1 Stalagmite δ^{18}O Record From Turkey[a]

Event	Timing NGRIP (Years Before 2000 A.D.)	Timing Sofular (Years Before 2006 A.D.)
0	11700	-
1	14680	(14658)
2	23340	-
3	27780	28017
4	28900	29436
5	32500	32590
6	33740	33931
7	35480	35943
8	38220	38173
9	40160	40303
10	41460	41736
11	43340	44169
12	46860	47675
13	49280	-
14	54220	-
15	55800	-
16	58280	-
17	59440	-

[a]The values for the NGRIP ice core are taken from the publications of *Ditlevsen et al.* [2007] and *Svensson et al.* [2008]. For the Sofular record, the values are taken from the auxiliary material of *Fleitmann et al.* [2009]. Note that the continuous part of the Sofular record only allows estimation of the timing of the events 3–12 since the stalagmite was reported to stop growing at some time interval in between DO events 3 and 1. Therefore, we only use the timing of the events 3–12 in our study. Detailed discussions of the dating uncertainties are given by *Andersen et al.* [2006] and *Ditlevsen et al.* [2007], and *Fleitmann et al.* [2009].

distribution) has not yet been statistically rejected on the basis of the NGRIP or Sofular records, and also not on the basis of the GISP2 ice core record (apart from the case where the DO event 9 is removed from the record) [*Ditlevsen et al.*, 2007]. This further justifies the use of the geometric distribution as the null hypothesis in our study.

[7] Figure 2 (top) shows the mean value of the simulated interevent waiting time as a function of the noise intensity σ_n, where each data point corresponds to a simulation containing 2,000,000,000 years. Note that in the relevant range of millennial-scale waiting times, this mean value is expected to be very close to the population mean interevent waiting time of the simulated events, due to the large number of events in the Monte Carlo simulation (200,000 to 2,000,000). The dashed respectively dotted lines label the values as obtained from the sample of 10–18 events as seen in the two paleoclimatic proxy records. The timing of the corresponding events is given in Table 1. We note that in the case of the Sofular record, we only use the continuous part (including the events 3–12), but not the event 1, since the stalagmite was reported to stop growing at some time interval in between the events 3 and 1 [*Fleitmann et al.*, 2009]. By assuming that the sample mean interevent waiting time as obtained from the two proxy records equals the population mean interevent waiting time as obtained from our Monte Carlo simulations, we estimate the following values of the standard deviation σ_n in the framework of our random process: σ_n = 0.408 (NGRIP) and σ_n = 0.424 (Sofular) (see Figure 2). Figure 2 (bottom) shows the ratio between the simulated mean absolute deviation and the simulated mean value of the interevent waiting time, as a function of the simulated mean waiting time. Since p is small, this ratio is rather insensitive to the choice of σ_n and is close to 2/e, as expected from equations (4) and (5).

[8] In order to test if the waiting time histogram of the most recent DO events as obtained from the two paleoclimatic proxy records, whose timing is given in Table 1, is statistically consistent with a simple geometric distribution, we perform a Monte Carlo based null hypothesis test (Figure 3). The standard procedure in null hypothesis testing is as follows [*Lehmann and Romano*, 2005; *Ditlevsen et al.*, 2007; *Mudelsee*, 2010]: To explain a given data set, a null hypothesis H0 is formulated together with an alternative, mutually exclusive hypothesis H1. As described above, our null hypothesis is that DO events are generated by a geometric distribution. An alternative hypothesis H1 is described in section 3. It should be noted that, of course, both H0 and H1 should represent plausible hypotheses from a geological, geophysical and dynamical system viewpoint. A test statistic is then chosen, whose value is calculated from the given data set. Depending on its value and its distribution under H0 ("null distribution"), either H0 is not rejected or H0 is rejected in favor of H1. Of course, the chosen test statistic should have noteworthy statistical power to distinguish between H0 and H1. We use Monte Carlo simulations to estimate the null distribution. In our simulations, we calculate the sample mean absolute deviation of the interevent waiting time τ from its sample mean value $\Delta\tau$ (Figure 3), divided by the sample mean interevent waiting time $\Delta\tau$. Our motivation for choosing this test statistic is that this property, when calculated from the sample of observed DO events, does not contain free parameters, which is desired. Moreover, since many researchers regard DO events as being somewhat regular [*Alley et al.*, 2001; *Schulz*, 2002; *Rahmstorf*, 2003], we think that a measure of regularity should be used in our approach too. Our test statistic represents such a measure, since it takes a minimum value of zero in the hypothetic case of perfectly periodic events. For the 18 most recent DO events in the NGRIP record, as given in Table 1, our test statistic yields a value of 0.473 (sample mean absolute deviation: 1328 years, sample mean: 2808 years). We then simulate the distribution of this measure as obtained under the null hypothesis H0 that the events in the ice core record follow a geometric distribution (with parameter σ_n = 0.408, compare Figure 2). From the simulation we find that values of 0.473 or lower only occur in 1.7 percent of the cases (ensemble size: 40,000 realizations) (compare Figure 3). In addition to that, using the Kolmogorov-Smirnov test, we also find that it is possible to reject H0 (with parameter σ_n = 0.408) at the 0.05 significance level.

[9] Finally, we also use the timing of DO events as obtained from the Sofular record in an attempt to reject H0 (with parameter σ_n = 0.424, compare Figure 2). For the DO events 3–12 in the Sofular record, as given in Table 1, our test statistic yields 0.263 (sample mean absolute deviation: 575 years, sample mean: 2184 years). From our Monte Carlo simulation (Figure 3) we find that values of 0.263 or lower only occur in 1.4 per mille of the cases (ensemble size: 70,000 realizations). In addition to that, using the Kolmogorov-Smirnov test, we also find that it is possible to

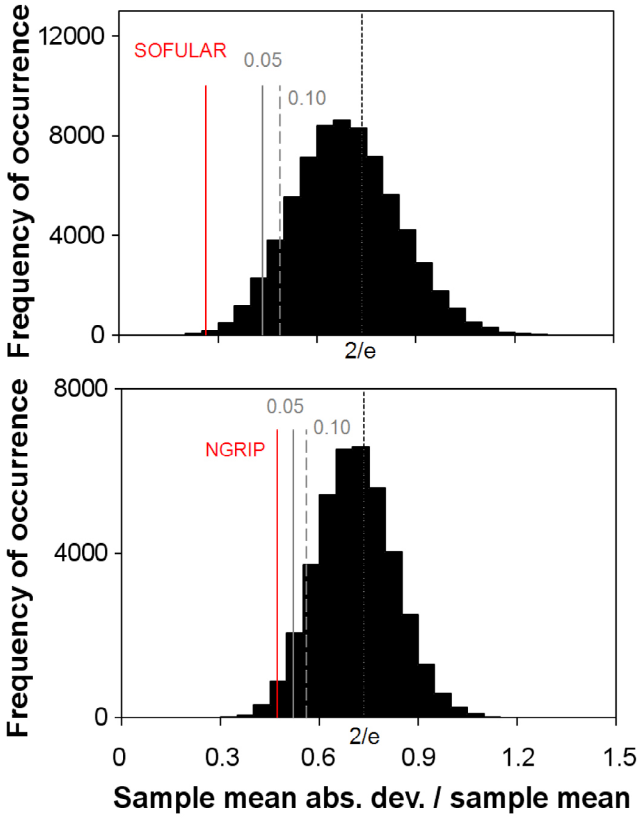

Figure 3. Results of the Monte Carlo simulation. The distribution of the test statistic $\langle|\tau - \Delta\tau|\rangle/\Delta\tau$, that is, of the ratio between (1) the sample mean absolute deviation ($\langle|\tau - \Delta\tau|\rangle$) of the interevent waiting time τ from the sample mean $\Delta\tau$ and (2) the sample mean $\Delta\tau$, as obtained from the considered random process H0 (geometric distribution) is shown. The value of the test statistic as calculated from the sample of the 10 events (3–12) as recorded (top) in the continuous part of the Sofular δ^{18}O record of the 18 events (0–17), respectively, and (bottom) in the NGRIP δ^{18}O record is depicted by the red lines. The gray lines indicate the 10% (dashed) and 5% (solid) significance limits (one sided test). The dotted black lines show the value of $\langle|\tau - \langle\tau\rangle|\rangle/\langle\tau\rangle$, that is, the ratio between the population mean absolute deviation ($\langle|\tau - \langle\tau\rangle|\rangle$) and the population mean value $\langle\tau\rangle$. This value is close to 2/e, compare Figure 2 (bottom). Note that the maximum of the test statistic distribution does not exactly equal $\langle|\tau - \langle\tau\rangle|\rangle/\langle\tau\rangle$, since the sample mean absolute deviation as averaged over all ensemble members of the Monte Carlo simulation is not an unbiased estimator of the population mean absolute deviation [*Triola*, 2005]. The ensemble size is 40,000 (NGRIP) and 70,000 (Sofular) realizations, respectively. For more information, see text.

reject H0 (with parameter σ_n = 0.424) at the 0.05 significance level, based on the Sofular data. We note that the main reason for the much higher statistical significance based on the Sofular record is that this record does not contain the DO event 2, since the stalagmite did not grow at that time. In contrast, the results obtained on the basis of the last 18 DO events in the NGRIP record are strongly influenced by the presence of one single, conspicuously long waiting time between the DO events 1 and 2. This interval coincides with the Last Glacial Maximum, i.e., with the epoch of presumably maximum ice sheet extent during the Last Glacial Period. This coincidence could point at nonstationarity as a possible explanation for this exceptionally long waiting time.

[10] We note that in a recent study, the null hypothesis of an exponential waiting time distribution (i.e., the continuous analog of the geometric distribution) was found to be consistent with the timing of the most recent DO events as seen in both the NGRIP and the GISP2 record, but was reported to be inconsistent with the case when the DO event 9 is removed from the GISP2 record [*Ditlevsen et al.*, 2007]. In contrast, we are now able to statistically reject the geometric distribution based on the NGRIP and Sofular records. Thus, this work is a new step for statistical analyses on the regularity of DO events and, possibly, for a better understanding of the generation of DO events.

3. H1: A Simple Two-Parameter Random Process

[11] To construct a more viable random process as an alternative hypothesis H1, we now generalize the process described in section 2 as follows: We abandon our assumption that the threshold value $T = 1$ is constant in time. Instead, we postulate a simple time-dependent threshold function $T(t)$ that follows a relaxation law after the onset of each DO event. This assumption is in first place motivated by dynamical system theory: When shifted into a nonequilibrium configuration by some perturbation, many dynamical systems approach their equilibrium state following a relaxation law with some characteristic time scale. For example, results obtained earlier with an ocean-atmosphere model of intermediate complexity [*Ganopolski and Rahmstorf*, 2001] illustrated that the onset of DO-like events in that model coincides with a strong overshooting in the concentration of salinity and heat in the area of deep convection of the northern North Atlantic [cf. *Ganopolski and Rahmstorf*, 2001, Figure 5]. This overshooting is apparently followed by a millennial-scale relaxation process back to preevent conditions. It was further reported that this implies a time-dependent stability of the simulated events (that is, the longer a simulated interstadial/stadial persists, the smaller a perturbation is required in order to trigger the transition back to the stadial/interstadial state [*Braun et al.*, 2007]), consistent with our assumption of a relaxation law in the threshold function $T(t)$. Finally, the assumption of a relaxation process during DO events is also supported by some geological climate proxy records: DO events as recorded e.g., in Greenland ice core records (Figure 1, top) appear to have a characteristic saw-tooth shape, with highest temperatures during the beginning of the events and gradually decreasing temperatures toward the end of the events, which is consistent with our assumption of the existence of a relaxation process.

[12] Now, let t_n denote the timing of the nth event. We assume that $T(t) = 1$ for $t = t_n$ ($n = 1, 2, 3, \ldots$) and that $T(t) = \exp(-[t - t_n]/\tau_0)$ for $t_n < t < t_{n+1}$. Here, the parameter τ_0 (with $\tau_0 > 0$) denotes the relaxation time of the random process. Note that $T(t)$ shows a discontinuity at the timing t_n of each event and afterwards follows an exponential law

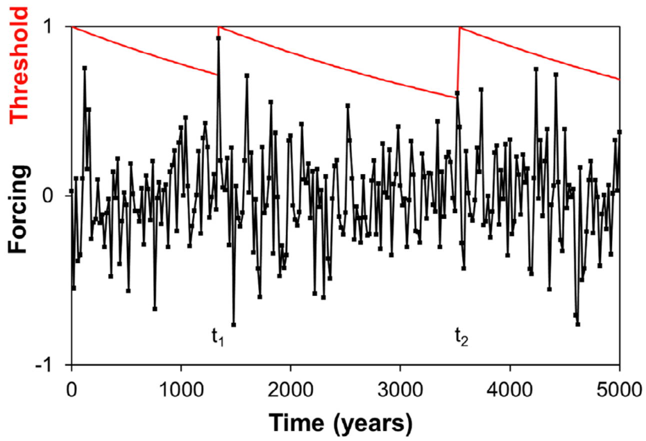

Figure 4. Dynamics of the two-parameter random process H1. The random input (black) together with the output, which is given by the time evolution of the threshold function $T(t)$ (red), is shown. Each time the forcing exceeds the threshold function, which happens at times t_1 and t_2, the threshold function takes a maximum value of one. Afterward, it approaches its time equilibrium value of zero following a relaxation process with time scale τ_0. Note that apart from the time step, which is chosen to be 20 years, the normalized random process has two parameters, that is, the relaxation time τ_0 and the standard deviation σ_n of the noise.

toward its time equilibrium value of zero (Figure 4). We further note that, apart from the time step, which we choose to be 20 years as in the case of the one-parameter process (H0), this normalized random process now has two parameters, that is, the standard deviation σ_n of the noise and the relaxation time τ_0. We also note that the two-parameter process (H1) and the one parameter process (H0) are mutually exclusive for any finite value of the parameters τ_0 and σ_n. But for a given standard deviation σ_n of the noise, H1 appears to approach H0 in the limit $\tau_0 \to \infty$.

[13] Having formulated the random process H1, the next step is to estimate the parameters σ_n and τ_0 based on the sample of 10–18 DO events as observed in the two paleoclimatic proxy records. Since we only have two free parameters, we apply the following simple estimation method, but we note that more efficient estimation procedures might exist: For many different combinations of the two parameters τ_0 and σ_n, we perform a Monte Carlo simulation of the process H1, from which we obtain the population mean value of the interevent waiting time ($\langle\tau\rangle$) and the ratio between (1) the population mean absolute deviation from the population mean and (2) the population mean value of the interevent waiting time ($\langle|\tau - \langle\tau\rangle|\rangle/\langle\tau\rangle$). We then plot isolines of these two properties, as a function of τ_0 and σ_n (Figure 5). The point of intersection of the two isolines that correspond to the values of $\langle\tau\rangle$ and $\langle|\tau - \langle\tau\rangle|\rangle/\langle\tau\rangle$ as obtained from the sample of events in the two proxy records is our estimate of the values of τ_0 and σ_n in the framework of our process H1. In this way, we obtain τ_0 = 3980 years and σ_n = 0.284 for the Sofular record, and τ_0 = 15,840 years and σ_n = 0.355 for the NGRIP record. Again we note that in the case of the NGRIP record, the estimate is strongly influenced by the existence of one conspicuously long waiting time between the DO events 1 and 2, which may be the result of nonstationarity. Note that, according to our method, only one unique parameter combination exists for each paleoclimatic proxy record (Figure 5). We also note that the estimated values of τ_0 and σ_n are obtained for one particular (albeit plausible) choice of the time step, and that a different choice of the time step could lead to different parameter values. Figure 5 (bottom) demonstrates that, with the estimated values of the parameters τ_0 and σ_n, the process H1 matches the waiting time histogram of DO events as seen in the proxy records to a good approximation. Note that in the NGRIP record, the longest waiting time between the events 1 and 2 of about 8700 years still falls within the tail of the generated distribution. We further tested by means of the Kolmogorov-Smirnov test that the generated distributions of the interevent waiting times are consistent with the waiting time histogram of the events in the proxy records, even at a significance level of 0.2. Compared with a simple geometric distribution (H0), which we find to be statistically inconsistent with the recurrence pattern of DO events (section 2), the waiting time distribution as generated by the process H1 therefore better fits the waiting time histogram of DO events. Thus, the applied process H1 seems to represent a more realistic stochastic process for the generation of random DO events and consequently a more adequate null hypothesis for future Monte Carlo based statistical analyses on the recurrence pattern of DO events.

[14] As a final remark, we would like to stress that several other processes exist which were used to simulate the waiting time distribution of DO events, e.g., a six-parameter model, which was constructed from the dynamics of DO events as seen in an ocean-atmosphere model of intermediate complexity, and whose parameter values were obtained by an intuitive fit with that model [*Braun et al.*, 2007]. The main strength of our two-parameter process as compared with that model is its simplicity: For example, the smaller number of adjustable parameters enables us to perform a much more systematic parameter estimation procedure, in order to fit the model-generated interevent waiting time distribution to the waiting time histogram of DO events as seen in the proxy records (Figure 5). In addition to that, our process is in first place based on the principle of parsimony and on – according to our interpretation – very plausible principles of nonequilibrium dynamics, and not in first place on a fit with an ocean-atmosphere model. Hence, this is a new line of reasoning in order to infer about the generation of DO events, which is largely independent of assumptions made in ocean-atmosphere models.

4. H2: An Added Bisinusoidal Cycle (Mimicking Solar Forcing)

[15] Finally, we apply the proposed random process H1 for a case study, in which we investigate how an added bisinusoidal periodic forcing, which mimics two reported century-scale solar activity cycles, alters the distribution of the output events as simulated by this random process. This yields another process, H2. We explicitly note, however, that we here apply the principle of parsimony and we thus do not advocate the process H2 as being the generating process of DO events: As we discuss later in this section, already in a highly simplified solar forcing scenario a

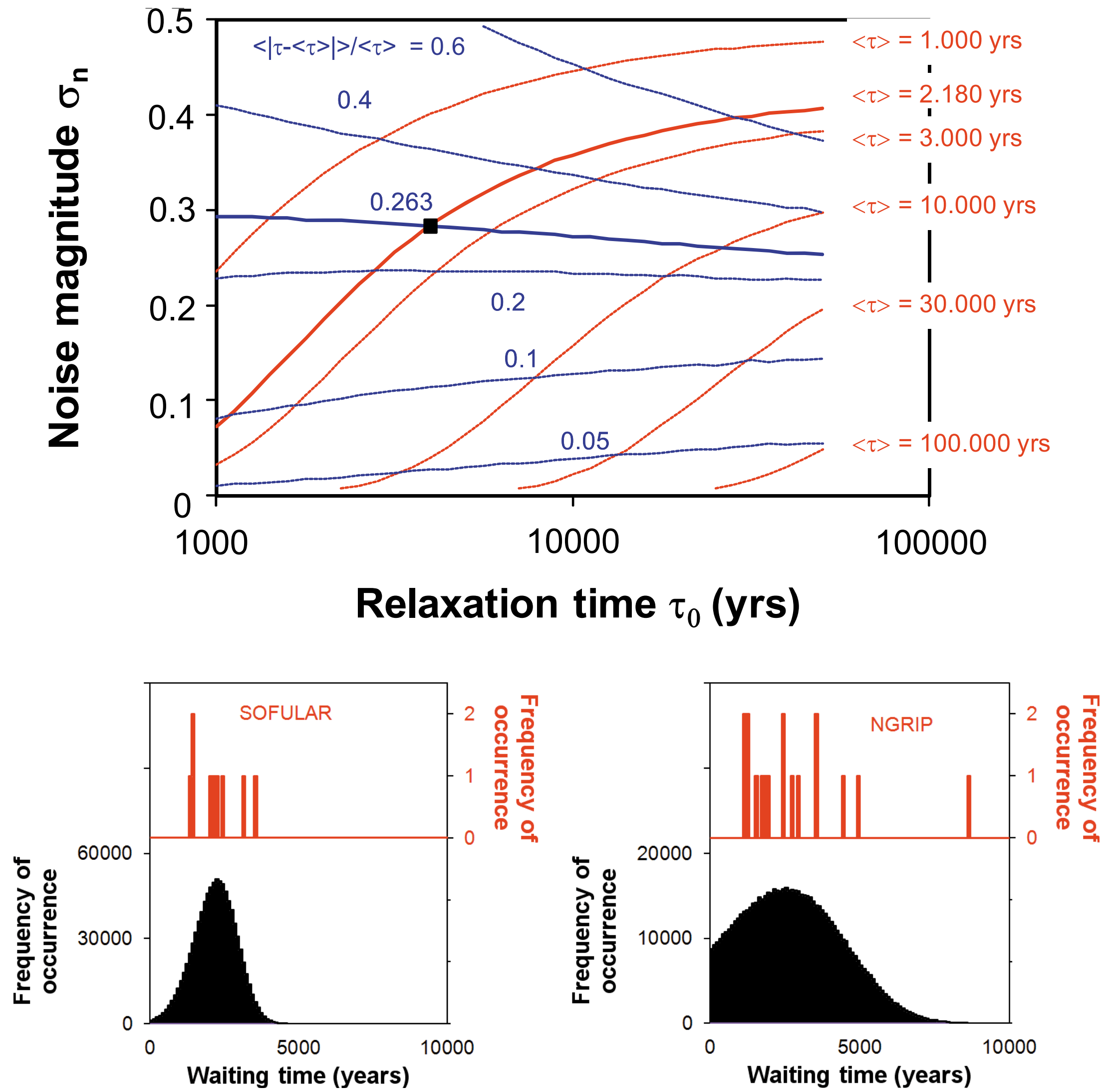

Figure 5. Parameter estimation of the process H1. (top) Isolines of constant population mean interevent waiting times $\langle\tau\rangle$ (red) and of constant ratios between the population mean absolute deviation and the population mean interevent waiting time, $\langle|\tau - \langle\tau\rangle|\rangle/\langle\tau\rangle$ (blue), as a function of the parameters τ_0 and σ_n. Thick lines correspond to the values as obtained from the sample of 10 DO events (3–12) in the Sofular isotopic record (Table 1). The point of intersection, which is marked in black, represents our estimation of the parameters τ_0 and σ_n (see text for more details). This approach yields $\tau_0 \approx 4000$ years and $\sigma_n \approx 0.284$ for the Sofular record, and $\tau_0 \approx 16{,}000$ years and $\sigma_n \approx 0.355$ for the NGRIP record. For each point, the length of the simulation is 10,000,000 time steps (i.e., 200,000,000 years). (bottom) The waiting time distribution as generated by the process H1 (black), using the values of the parameters τ_0 and σ_n as obtained by our parameter estimation procedure. In addition to that, the histogram of the interevent waiting times as obtained from the (left) Sofular and (right) NGRIP records is also shown (red). The binning is 100 years. The ensemble size of the simulations in Figure 5 (bottom) is about 800,000 events.

complex, nonperiodic recurrence pattern of DO events could be expected. Thus, we are not aware of the existence of any study in which a statistically significant regularity in the timing of DO events was demonstrated that gives noteworthy support for a possible solar influence on the events.

[16] Various studies hypothesized about a possible solar role in triggering DO events. Several argument could be or have been used in favor of a possible solar influence on the timing of DO events:

[17] 1. Small (~0.1%) cyclic solar irradiance variations have been measured by satellites over the last three decades, and decadal-to-century scale cyclic solar variations have further been observed by sunspot counting over the last few centuries [*Gleissberg*, 1944]. Moreover, century-scale cyclic variations were reported to exist in a 1000 year long record of solar-terrestrial phenomena [*Feynman and Fougere*, 1984]. Finally, century-scale cyclic solar proxy variations were also reported to persist throughout the entire Holocene [*Stuiver and Braziunas*, 1993; *Peristykh and Damon*, 2003]

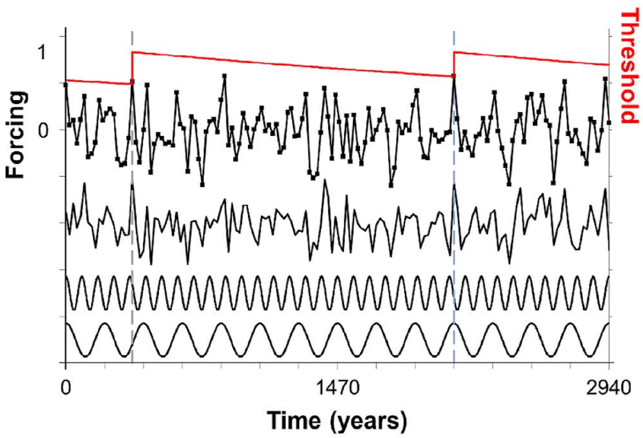

Figure 6. Addition of two sinusoidal input cycles, which mimic the leading spectral components of the reported solar Gleissberg (~88 year) and De Vries/Suess (~208 year) cycles, respectively. The input (black) together with the output of the process H2, which is given by the time evolution of the threshold function $T(t)$ (red), is shown as in Figure 4. The top black curve shows the combined input, which consists of (1) a Gaussian-distributed random component with zero mean and standard deviation σ_n (second black curve from top), and (2) two sinusoidal components with equal amplitudes $A = A_1 = A_2$, which are chosen to be identical to the standard deviation σ_n of the random component, and with periods of 1470/17 (≈86.5) and 1470/7 (=210) years, respectively (bottom black curves). The dashed lines indicate the beginning of the DO-like events, which are triggered each time the combined forcing crosses the threshold function $T(t)$. Note that due to the presence of a random component in the input, the output events do not exhibit a stable phase relationship with the two sinusoidal input cycles and are also not always triggered during intervals of particularly large excursions of the bisinusoidal input. For more information, see text.

and throughout the second half of the Last Glacial Period [*Wagner et al.*, 2001]. The apparent presence of this forcing could affect the timing of DO events, since in a system with a threshold already small perturbations could push the system above the threshold.

[18] 2. A close correlation between multicentennial climate proxy anomalies in the North Atlantic and "rapid (100–200 year), conspicuously large-amplitude variations" in solar proxy records was reported to exist throughout the Holocene. This was interpreted as an indication for persistent solar influence on North Atlantic climate, at least during the Holocene [*Bond et al.*, 2001].

[19] In contrast, several other hypotheses for the timing of DO events were proposed that are in first place based on the existence of apparent regularities in the recurrence pattern of DO events, the statistical significance of them depends on the applied null hypothesis and on the chosen test statistic and which could be statistically insignificant [*Ditlevsen et al.*, 2007; *Braun et al.*, 2010]. Hence, the hypothesis of a possible solar role in triggering DO events could thus be based in first place on arguments that are independent from any apparent regularity in the timing of DO events. This is desirable for null hypothesis testing, since it prevents circular reasoning. We explicitly note, though, that a possible solar influence on the timing of DO events does not necessarily conflict with a leading role of other factors, such as random variability [*Ditlevsen et al.*, 2007], in triggering the events, since the events could have been triggered by a combination of several factors [*Braun et al.*, 2008]. We further note that various arguments could be or have been used against a possible solar role in triggering DO events:

[20] 1. The magnitude of solar irradiance variations as recorded by satellites over the last three decades is small (i.e., only about 0.1% of the "solar constant"), which is often regarded as being orders of magnitude too small to trigger 10 to 15 K DO-like climatic anomalies.

[21] 2. No noteworthy correlation and no stable phase relation were reported to exist between cyclic century-scale solar proxy variations and DO events [*Muscheler and Beer*, 2006]. In addition to that, only some DO events were reported to begin during intervals of particularly large solar proxy variations [*Muscheler and Beer*, 2006].

[22] 3. A prominent 1470 year spectral component, which is closely related to the occurrence of DO events and whose statistical significance is still under debate, since it could depend heavily on the applied null hypothesis [*Schulz*, 2002; *Braun et al.*, 2010], was reported to exist in the GISP2 ice core δ^{18}O record [*Grootes and Stuiver*, 1997]. Proxies of solar variability, in contrast, were reported to exhibit prominent spectral peaks especially at periods of about 208 and 88 years and at harmonics, respectively combination tones thereof [*Stuiver and Braziunas*, 1993; *Peristykh and Damon*, 2003], but not at a period of about 1470 years. However, in a highly nonlinear system a spectral correlation between the input and the output is not necessarily expected, and it was explicitly demonstrated with a coupled ocean-atmosphere model of intermediate complexity that DO-like events, spaced periodically by 1470 years (or, multiples thereof), can be triggered by a periodic input in cycles of 210 and ~86.5 years but without any input power at a spectral component corresponding to a period of 1470 years [*Braun et al.*, 2005].

[23] In this light, we think that it is of relevance to investigate how the inclusion of century-scale forcing cycles, which mimic the leading spectral components of the reported De Vries/Suess (~208 year) [*Wagner et al.*, 2001; *Peristykh and Damon*, 2003] and Gleissberg (~88 year) [*Feynman and Fougere*, 1984; *Peristykh and Damon*, 2003] solar cycles, may alter the waiting time distribution and phase distribution of noise-induced DO-like events according to our random process H1. As before, we thus drive the process H1 by Gaussian-distributed noise, with zero mean and standard deviation σ_n. We then add a periodic forcing component, consisting of two sinusoidal cycles with equal amplitudes $A = A_1 = A_2 = \sigma_n > 0$, which are chosen to be equal to the standard deviation of the noise, and with periods T_1 = 1470/7 (=210) years and T_2 = 1470/17 (≈86.5) years (Figure 6). We note that our choice of T_1 and T_2 is a very particular (but, considering the uncertainties in the solar cycle "periods," possible) one. Our motivation for choosing these values is that this particular input scenario is considerably simpler to test by means of null hypothesis testing, because the bisinusoidal input repeats periodically in this scenario (with a period of 1470 years), which implies a maximum regularity of the DO-like output events, as we

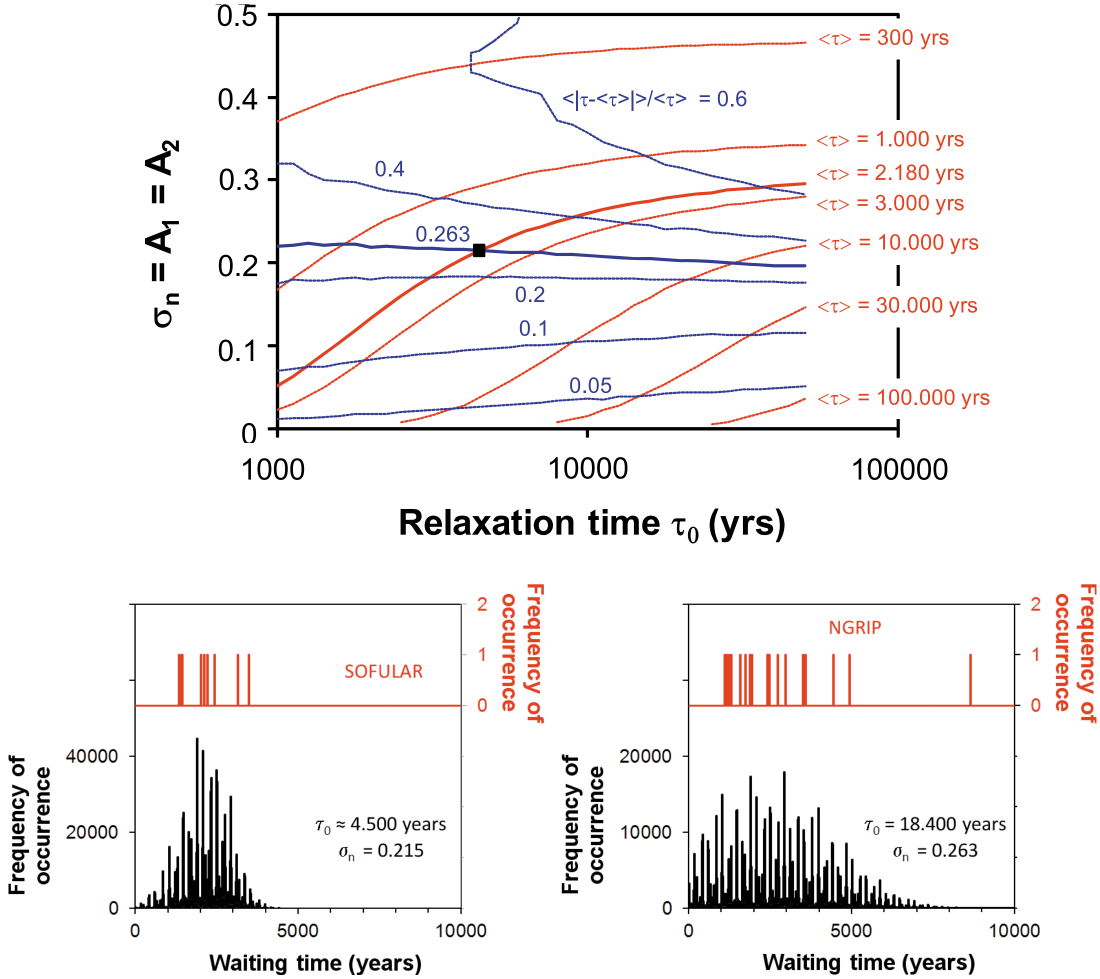

Figure 7. Parameter estimation of the process H2. (top) Isolines of constant population mean interevent waiting times $\langle\tau\rangle$ (red) and of constant ratios between the population mean absolute deviation and the population mean interevent waiting time, $\langle|\tau - \langle\tau\rangle|\rangle/\langle\tau\rangle$ (blue), as a function of the parameters τ_0 and σ_n (with $\sigma_n = A_1 = A_2$). Thick lines correspond to the values as obtained from the sample of 10 DO events (3–12) in the Sofular isotopic record (Table 1). The point of intersection, which is marked in black, corresponds to our estimation of the parameters τ_0 and σ_n (see text for more details). This approach yields $\tau_0 \approx 4500$ years and $\sigma_n = A_1 = A_2 \approx 0.215$ for the Sofular record, and $\tau_0 \approx 18{,}400$ years and $\sigma_n = A_1 = A_2 \approx 0.263$ for the NGRIP record. For each point, the length of the simulation is at least 10,000,000 time steps (i.e., 200,000,000 years). (bottom) The waiting time distribution as generated by the process H2 (black), using the values of the parameters as obtained by our approach. In addition to that, the histogram of the interevent waiting times as obtained from the (left) Sofular and (right) NGRIP records is also shown (red). The binning is 20 years. The ensemble size of the simulations in Figure 7 (bottom) is about 800,000 events.

will discuss later. In the case of a nonperiodic input, in contrast, a more irregular recurrence pattern of the DO-like events could be expected, which would be more difficult to analyze by means of null hypothesis testing, because measures of regularity do not necessarily have a noteworthy power to distinguish between a perfectly random occurrence of DO events and an irregular but nonrandom one. We further stress that we do not suggest that the reported solar cycles are sinusoidal. We are only using a sinusoidal input for simplicity. We also note that the possible solar origin of the reported De Vries/Suess (~208 year) cycle, and of possible longer cycles [*Damon and Sonett*, 1991], is less well established than of the reported Gleissberg (~88 year) cycle, because it is more difficult to relate these cycles to features in the historical sunspot record [*Wagner et al.*, 2001], although additional arguments for the possible existence of a ~208 year solar cycle were given [*Stuiver and Braziunas*, 1993; *Wagner et al.*, 2001; *Peristykh and Damon*, 2003], the quality of which is debatable. Moreover, we note that we consider only two input cycles, because measures of regularity are likely to have a higher statistical power to distinguish between the scenarios H1 and H2 than between H1 and a possible scenario with more than two input cycles. Finally, we also note that the proposed scenario (H2) and the previous scenario (H1) are mutually exclusive for any nonzero value of the parameter $\sigma_n = A_1 = A_2 = A$. As

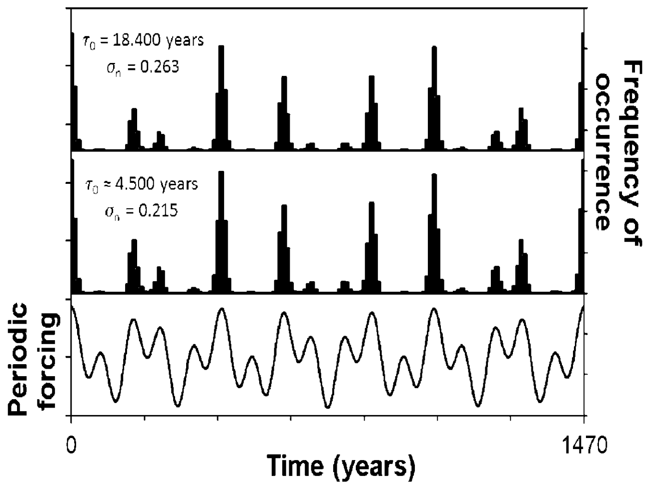

Figure 8. Phase distribution of the DO-like events, as obtained from the process H2. The bottom curve shows the bisinusoidal input component, with a period of 1470 years. The top curve shows the 1470 year phase distribution of the onset of the simulated events, that is, the timing of the onset modulo 1470 years. Note that this distribution has several modes, which correspond to local maxima of the bisinusoidal input. The number of modes and their magnitude depends on the signal-to-noise ratio in the forcing, compare Figures 5 and 6 of *Braun et al.* [2009]. The sample size is approximately 800,000 events. The chosen parameters values are the ones we obtained with our parameter estimation approach as illustrated in Figure 7, that is, $\tau_0 \approx 18{,}400$ years and $\sigma_n = A_1 = A_2 \approx 0.263$ (top), respectively $\tau_0 \approx 4500$ years and $\sigma_n = A_1 = A_2 \approx 0.215$ (bottom).

before, the time step is chosen to be 20 years, in accordance with the two applied paleoclimatic records.

[24] In order to estimate the values of the two parameters σ_n and τ_0 of the process H2 from the waiting time histogram of the DO events as seen in the two climate proxy records, we again follow an isoline approach (Figure 7, top), analogous to the one applied in Figure 5. As before, we estimate the parameter values from the point of intersection of the two isolines whose values of $\langle\tau\rangle$ and $\langle|\tau - \langle\tau\rangle|\rangle/\langle\tau\rangle$ agree with the corresponding sample mean values as obtained from the 10–18 events in the two proxy records. This approach yields σ_n = 0.215 and $\tau_0 \approx 4500$ years for the Sofular record, and σ_n = 0.263 and τ_0 = 18400 years for the NGRIP record. Again we mention that these values are obtained only for our particular choice of the time step. As before, we note that the obtained parameter values for the NGRIP record are strongly influenced by one single, conspicuously long waiting time between the DO events 1 and 2, which may be the result of nonstationarity.

[25] With this estimation procedure of the parameter values σ_n and τ_0, the generated waiting time distributions agree fairly well with the waiting time histograms of the DO events as seen in the two proxy records (Figure 7, bottom): Using the Kolmogorov-Smirnov test, we find that we cannot reject H2 on the basis of the waiting time histograms of the DO events as seen in the two paleoclimatic proxy records, not even at a significance level of 0.2. Note that the waiting time distributions of the DO-like events as obtained from the process H2 exhibit many modes, which are typically spaced by only 100–200 years. These modes arise from the presence of many local maxima in the bisinusoidal input, which are typically also spaced by 100–200 years (Figure 6). Similarly, the phase distribution of the output events likewise exhibits several modes, which coincide with local maxima in the bisinusoidal forcing component (Figure 8). Distinguishing between the processes H1 and H2 on the basis of the waiting time distribution/phase distribution is thus difficult and requires measures of multimodality, according to our interpretation. We stress that so far measures of (quasi)periodicity have commonly been used to investigate the regularity of DO events [*Alley et al.*, 2001; *Schulz*, 2002; *Rahmstorf*, 2003]. Based on our findings as presented here, we do not expect that such measures have a noteworthy statistical power to distinguish between the processes H1 and H2. We thus advocate using nonparametric measures of nonperiodic regularity, for example of multimodality [*Silverman*, 1981; *Fischer et al.*, 1994; *Minnotte*, 1997], to distinguish between a random occurrence of DO events and a scenario in which the events are at least to some part triggered by solar variability. For completeness we note that at least one study exists in which the recurrence pattern of DO events in the most recent part of the NGRIP ice core δ^{18}O record was analyzed using a parametric measure of multimodality [*Braun et al.*, 2009]. However, Braun et al. performed no estimation of the statistical significance of the detected recurrence pattern because of the difficulty to come up with (1) a simple but realistic stochastic process for the occurrence of random DO events and (2) a simple but powerful nonparametric measure of multimodality. Thus, our present study solves the first of these two problems and presents a relevant step toward identifying the trigger of the remarkable DO events in glacial climate.

5. Discussion and Conclusions

[26] In this paper we connected the statistical concept of hypothesis testing with the principle of parsimony in an approach to analyze the waiting time statistics of DO events. Our study provides three main results, which are summarized in Figure 9.

[27] 1. We constructed a one-parameter nonlinear stochastic process (H0), in which DO events are triggered each time a random forcing crosses a fixed threshold value. This process leads to a geometric distribution of the interevent waiting times (Figure 9), respectively to an exponential distribution, which is the continuous analog of the geometric distribution and which has so far not been rejected [*Ditlevsen et al.*, 2007]. Using the timing of the most recent DO events as seen in two paleoclimatic proxy records, the NGRIP ice core δ^{18}O record and the Sofular cave stalagmite δ^{18}O record (Figure 9), we now find that the recurrence pattern of the most recent DO events is inconsistent with a geometric distribution, and we can statistically reject the geometric distribution at the 0.05 (NGRIP) respectively 0.01 (Sofular) significance level. Thus, our study indicates that there is a need to come up with a more realistic stochastic process for the occurrence of random DO events. We attribute the different outcome of our present study as compared with the findings of *Ditlevsen et al.* [2007] to the fact that

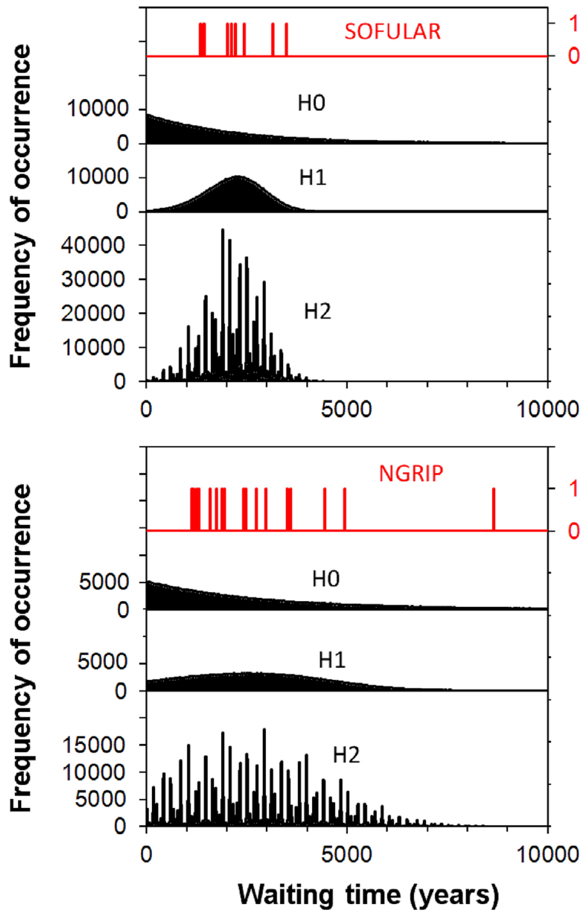

Figure 9. Waiting time histogram of DO events as seen in the two considered paleoclimatic proxy records and as obtained from the three considered dynamical processes. The red histograms show the distribution of the interevent waiting times as obtained (top) from the DO events 3–12 in the Sofular cave So-1 stalagmite δ^{18}O record and (bottom) from the DO events 0–17 in the NGRIP ice core δ^{18}O record. The timing of these events is given in Table 1. The black histograms show the distribution of the interevent waiting times as obtained from the dynamical processes H0, H1 and H2. The parameters of these processes are the ones given in Figures 2, 5 and 7, that is, $\sigma_n = 0.424$ (H0), $\tau_0 \approx 4000$ years and $\sigma_n \approx 0.284$ (H1), and $\tau_0 \approx 4500$ years and $\sigma_n = A_1 = A_2 \approx 0.215$ (H2) for the fit with the Sofular data (Figure 9, top), respectively $\sigma_n = 0.408$ (H0), $\tau_0 \approx 16{,}000$ years and $\sigma_n \approx 0.355$ (H1), and $\tau_0 \approx 18{,}400$ years and $\sigma_n = A_1 = A_2 \approx 0.263$ (H2) for the fit with the NGRIP data (Figure 9, bottom). In the black histograms, the sample size is approximately 800,000 events. The binning is 20 years.

the main difference between the DO-like events as obtained from the processes H1 (see below) and H0 is the dispersion of the interevent waiting time distribution (Figure 9), and not the phase coherence of the events. We thus expect that our measure of regularity, which essentially is a measure of dispersion, has a higher statistical power to distinguish between H0 and H1 than the Rayleigh R statistic used by *Ditlevsen et al.* [2007], which is a measure for the phase coherence of the events.

[28] 2. We constructed a two-parameter nonlinear stochastic process (H1), in which DO events are triggered each time a random forcing crosses a time-dependent threshold function that follows a relaxation process with some characteristic time scale. Using the timing of the most recent DO events as seen in the NGRIP and Sofular δ^{18}O records, we find that this process is consistent with the waiting time histogram of the events as seen in both proxy records (Figure 9). Thus, we suggest to use our process H1 in future Monte Carlo based statistical tests on the regularity of DO events, for example to distinguish between a random occurrence of the events and a scenario in which the events are at least in part triggered by solar variability. Hence, our study provides a novel and relevant step toward identifying what triggered DO events in glacial climate.

[29] 3. Using our stochastic process H1, we investigated how the inclusion of a simple bisinusoidal forcing component, which mimics the leading spectral components of two reported century-scale solar cycles, affects the waiting time distribution and the phase distribution of the output events. This results in a third process, H2. We found that the additional periodic forcing component leads to the occurrence of several modes in the interevent waiting time distribution (Figure 9), respectively in the phase distribution (Figure 8). According to our interpretation, standard measures of periodicity thus do not have a noteworthy statistical power to distinguish between the processes H1 and H2 and should thus not be used to distinguish between a random occurrence of DO events (H1) and a scenario in which the events are at least in part triggered by solar variability. Instead, simple but powerful nonparametric measures of multimodality should be used, which are much more difficult to construct and apply. This approach should thus provide a novel and relevant step in the ongoing discussion about the possibility of a noteworthy solar contribution to triggering DO events.

[30] **Acknowledgments.** H. B. was funded by the German Research Foundation (DFG), project BR 3911/1-1. J. K. was funded by ECONS (WGL). The authors thank two anonymous referees for constructive comments that helped to increase the quality of the paper substantially.

References

Alley, R. B., S. Anandakrishnan, and P. Jung (2001), Stochastic resonance in the North Atlantic, *Paleoceanography*, *16*, 190–198, doi:10.1029/2000PA000518.

Andersen, K., et al. (2006), The Greenland Ice Core Chronology 2005, 15–42 ka. Part 1: Constructing the time scale, *Quat. Sci. Rev.*, *25*, 3246–3257, doi:10.1016/j.quascirev.2006.08.002.

Bond, G., W. Broecker, S. Johnsen, J. McManus, L. Labeyrie, J. Jouzel, and G. Bonani (1993), Correlations between climate records from North Atlantic sediments and Greenland ice, *Nature*, *365*, 143–147, doi:10.1038/365143a0.

Bond, G., B. Kromer, J. Beer, R. Muscheler, M. N. Evans, W. Showers, S. Hoffmann, R. Lotti-Bond, I. Hajdas, and G. Bonani (2001), Persistent solar influence on North Atlantic climate during the Holocene, *Science*, *294*, 2130–2136, doi:10.1126/science.1065680.

Braun, H., M. Christl, S. Rahmstorf, A. Ganopolski, A. Mangini, C. Kubatzki, K. Roth, and B. Kromer (2005), Possible solar origin of the 1,470-year glacial climate cycle demonstrated in a coupled model, *Nature*, *438*, 208–211, doi:10.1038/nature04121.

Braun, H., A. Ganopolski, M. Christl, and D. R. Chialvo (2007), A simple conceptual model of abrupt glacial climate events, *Nonlinear Processes Geophys.*, *14*, 709–721, doi:10.5194/npg-14-709-2007.

Braun, H., P. Ditlevsen, and D. R. Chialvo (2008), Solar forced Dansgaard-Oeschger events and their phase relation with solar proxies, *Geophys. Res. Lett.*, *35*, L06703, doi:10.1029/2008GL033414.

Braun, H., P. Ditlevsen, and J. Kurths (2009), New measures of multimodality for the detection of a ghost stochastic resonance, *Chaos*, *19*, 043132, doi:10.1063/1.3274853.

Braun, H., P. Ditlevsen, J. Kurths, and M. Mudelsee (2010), Limitations of red noise in analysing Dansgaard-Oeschger events, *Clim. Past*, *6*, 85–92, doi:10.5194/cp-6-85-2010.

Broecker, W. S., D. M. Peteet, and D. Rind (1985), Does the ocean-atmosphere system have more than one stable mode of operation?, *Nature*, *315*, 21–26, doi:10.1038/315021a0.

Damon, P. E., and C. P. Sonett (1991), Solar and terrestrial components of the atmospheric ^{14}C variation spectrum, in *The Sun in Time*, edited by C. P. Sonett, M. S. Giampapa, and M. S. Matthews, pp. 360–388, Univ. of Ariz. Press, Tucson.

Dansgaard, W., H. B. Clausen, N. Gundestrup, C. U. Hammer, S. Johnsen, P. M. Kristinsdottir, and N. Reeh (1982), A New Greenland deep ice core, *Science*, *218*, 1273–1277, doi:10.1126/science.218.4579.1273.

Ditlevsen, P., K. Andersen, and A. Svensson (2007), The DO-climate events are probably noise induced: Statistical investigation of the claimed 1,470 years cycle, *Clim. Past*, *3*, 129–134, doi:10.5194/cp-3-129-2007.

Feynman, J., and P. F. Fougere (1984), Eighty-eight year periodicity in solar-terrestrial phenomena confirmed, *J. Geophys. Res.*, *89*(A5), 3023–3027, doi:10.1029/JA089iA05p03023.

Fischer, N. I., E. Mammen, and J. S. Marron (1994), Testing for multimodality, *Comput. Stat. Data Anal.*, *18*(5), 499–512, doi:10.1016/0167-9473(94)90080-9.

Fleitmann, D., et al. (2009), Timing and climatic impact of Greenland interstadials recorded in stalagmites from northern Turkey, *Geophys. Res. Lett.*, *36*, L19707, doi:10.1029/2009GL040050.

Ganopolski, A., and S. Rahmstorf (2001), Simulation of rapid glacial climate changes in a coupled climate model, *Nature*, *409*, 153–158, doi:10.1038/35051500.

Gleissberg, W. (1944), A table of secular variations of the solar cycle, *Terr. Magn. Atmos. Electr.*, *49*, 243–244, doi:10.1029/TE049i004p00243.

Grootes, P. M., and M. Stuiver (1997), Oxygen 18/16 variability in Greenland snow and ice with 10^3- to 10^5-year time resolution, *J. Geophys. Res.*, *102*(C12), 26,455–26,470, doi:10.1029/97JC00880.

Grootes, P. M., M. Stuiver, J. W. C. White, S. Johnsen, and J. Jouzel (1993), Comparison of oxygen isotope records from the GISP2 and GRIP Greenland ice cores, *Nature*, *366*, 552–554, doi:10.1038/366552a0.

Johnson, N. L., S. Kotz, and A. W. Kemp (1992), *Univariate Discrete Distributions*, 2nd ed., John Wiley, New York.

Kapadia, C. H. (1983), Geometric distribution, in *Encyclopedia of Statistical Sciences*, vol. 3, edited by S. Kotz, N. L. Johnson, and C. B. Read, pp. 396–397, John Wiley, New York.

Lang, C., M. Leuenberger, J. Schwander, and S. Johnsen (1999), 16°C rapid temperature variation in central Greenland 70,000 years ago, *Science*, *286*, 934–937, doi:10.1126/science.286.5441.934.

Lehmann, E. L., and J. P. Romano (2005), *Testing Statistical Hypotheses*, 3rd ed., Springer, New York.

Minnotte, M. C. (1997), Nonparametric testing of the existence of modes, *Ann. Stat.*, *25*, 1646–1660, doi:10.1214/aos/1031594735.

Mudelsee, M. (2010), *Climate Time Series Analysis: Classical Statistical and Bootstrap Methods*, *Atmos. and Oceanogr. Sci. Library*, vol. 42, Springer, Dordrecht, Netherlands.

Muscheler, R., and J. Beer (2006), Solar forced Dansgaard/Oeschger events?, *Geophys. Res. Lett.*, *33*, L20706, doi:10.1029/2006GL026779.

Oeschger, H., J. Beer, U. Siegenthaler, B. Stauffer, W. Dansgaard, and C. C. Langway (1984), Late glacial climate history from ice cores, in *Climate Processes and Climate Sensitivity*, *Geophys. Monogr. Ser.*, vol. 5, edited by J. E. Hansen and T. Takahashi, pp. 299–306, AGU, Washington, D. C.

Peristykh, A. N., and P. E. Damon (2003), Persistence of the Gleissberg 88-yr solar cycle over the last 12,000 years: Evidence from cosmogenic isotopes, *J. Geophys. Res.*, *108*(A1), 1003, doi:10.1029/2002JA009390.

Rahmstorf, S. (2003), Timing of abrupt climate change: A precise clock, *Geophys. Res. Lett.*, *30*(10), 1510, doi:10.1029/2003GL017115.

Schulz, M. (2002), On the 1,470-year pacing of Dansgaard-Oeschger warm events, *Paleoceanography*, *17*(2), 1014, doi:10.1029/2000PA000571.

Severinghaus, J. P., and E. Brook (1999), Abrupt climate change at the end of the last glacial period inferred from trapped air in polar ice, *Science*, *286*, 930–934, doi:10.1126/science.286.5441.930.

Silverman, B. W. (1981), Using kernel density estimates to investigate multimodality, *J. R. Stat. Soc., B*, *43*, 97–99.

Spötl, C., and A. Mangini (2002), Stalagmite from the Austrian Alps reveals Dansgaard-Oeschger events during isotope stage 3: Implications for the absolute chronology of Greenland ice cores, *Earth Planet. Sci. Lett.*, *203*, 507–518, doi:10.1016/S0012-821X(02)00837-3.

Steffensen, J. P., et al. (2008), High-resolution greenland ice core data show abrupt climate change happens in few years, *Science*, *321*, 680–684, doi:10.1126/science.1157707.

Stuiver, M., and T. F. Braziunas (1993), Sun, ocean, climate and atmospheric CO_2: An evaluation of causal and spectral relationships, *Holocene*, *3*, 289–305, doi:10.1177/095968369300300401.

Svensson, A., et al. (2008), A 60000 year Greenland stratigraphic ice core chronology, *Clim. Past*, *4*(1), 47–57, doi:10.5194/cp-4-47-2008.

Triola, M. F. (2005), *Essentials of Statistics*, 2nd ed., Addison-Wesley, Boston, Mass.

Wagner, G., J. Beer, J. Masarik, R. Muscheler, P. W. Kubik, W. Mende, C. Laj, G. M. Raisbeck, and F. Yiou (2001), Presence of the solar de Vries cycle (~205 years) during the last ice age, *Geophys. Res. Lett.*, *28*(2), 303–306, doi:10.1029/2000GL006116.

Wang, Y. J., H. Cheng, R. L. Edwards, Z. S. An, J. Y. Wu, C.-C. Shen, and J. A. Dorale (2001), A high-resolution absolute-dated late Pleistocene monsoon record from Hulu Cave, China, *Science*, *294*, 2345–2348, doi:10.1126/science.1064618.

H. Braun, Heidelberg Academy of Sciences and Humanities, Im Neuenheimer Feld 229, D-69120 Heidelberg, Germany. (Holger.Braun@iup.uni-heidelberg.de)

P. Ditlevsen, Centre for Ice and Climate, Niels Bohr Institute, University of Copenhagen, Juliane Maries Vej 30, DK-2100 Copenhagen, Denmark.

J. Kurths, Potsdam Institute for Climate Impact Research, PO Box 60 12 03, 14412 Potsdam, Germany.

M. Mudelsee, Climate Risk Analysis, Schneiderberg 26, D-30167 Hannover, Germany.